Comments on

"Computation of the inverse of a polynomial matrix

and evaluation of its Laurent expansion"

by

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Int. J. Control, Vol. 53, No.2, 431-443.

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Abstract. The main purpose of this short comment is to note that the condition $p_{r,(r-1)q_1-v} \neq 0$ which is used for the computation of the Laurent expansion of $A(s)^{-1}$ is wrong. We propose therefore a correction to this statement.

Let

$$A(s) = A_0 + A_1 s + \cdots + A_{q_1} s^{q_1} \in \Re[s]^{r\times r}$$

with Smith form at $s = \infty$

$$S^{\infty}_{A(s)}(s) = \left(s^{q_1},\quad s^{q_2},\quad \cdots,\quad s^{q_k},\quad \frac{1}{s^{\hat{q}_{k+1}}},\quad \cdots,\quad \frac{1}{s^{\hat{q}_r}},\quad \mathbf{0}_{p-r,\,m-r} \right)$$

Consider also the determinant of A(s) which is given by the following polynomial

$$\det[A(s)] = (-1)^r p_r(s) = (-1)^r \sum_{k=0}^{rq_1} p_{r,k} s^k$$

(as has been defined by (3.5) and (3.16) in the above paper). Then in Corollary 3 of the above paper we have the following statement :

$$p_{r,(r-1)q_1-\hat{q}_r} \neq 0$$

Actually this condition is used for the computation of the inverse of the matrix

$$P = \begin{pmatrix} p_{r,(r-1)q_1-q_r} & 0 & \cdots & 0 & \cdots & 0 \\ p_{r,(r-1)q_1-q_r-1} & p_{r,(r-1)q_1-q_r} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ p_{r.0} & p_{r,1} & \cdots & p_{r,(r-1)q_1-q_r} & \cdots & 0 \\ 0 & p_{r.0} & \cdots & & & \vdots \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_{r.0} & \cdots & p_{r,(r-1)q_1-q_r} \end{pmatrix}$$

However as we can see from the first example of the same paper we may have $p_{r,(r-1)q_1-\hat{q}_r} = 0$ and so there exist no inverse of the matrix P so that to use it in the computation of the Laurent expansion of $A(s)^{-1}$.

Example 1. Let

$$A(s) = \begin{pmatrix} 1 & s^3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{pmatrix}$$

with Smith form at $s=\infty$

$$S^{C}_{A(s)}(s)=\begin{pmatrix} s^{3} & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & \frac{1}{s^{3}} \end{pmatrix}$$

and so $\mu=(r-1)q_1-\hat{q}_3=(3-1)\times 3-3=3$. However we can easily see that

$$\det[A(s)]=s\Rightarrow p_3(s)=-1\times s=p_{3,1}s$$

and thus $p_{3,3}=0$ which does not verify the corollary 3. •

Thus our suggestion is to find an integer k so that :

$$p_{r,(r-1)q_1-k}\neq 0$$

If this does not exitsts then the determinant of A(s) is zero and thus there will exist no inverse of A(s). In case where this integer exists then we shall have that the matrix

$$P=\begin{pmatrix} p_{r,(r-1)q_1-k} & 0 & \cdots & 0 & \cdots & 0 \\ p_{r,(r-1)q_1-k-1} & p_{r,(r-1)q_1-k} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ p_{r.0} & p_{r,1} & \cdots & p_{r,(r-1)q_1-k} & \cdots & 0 \\ 0 & p_{r.0} & \cdots & & & \vdots \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_{r.0} & \cdots & p_{r,(r-1)q_1-k} \end{pmatrix}$$

is invertible and so we shall compute the Laurent expansion of the inverse of A(s) according to the way proposed by the authors .

Also the result in Example 1 is wrong. The inverse of the matrix

$$A(s)=\begin{pmatrix} 1 & s^{3} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{pmatrix}$$

is

$$A(s)^{-1}=\begin{pmatrix} 1 & -s^{3} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{s} \end{pmatrix}$$

and not

$$A(s)^{-1} = \begin{pmatrix} 1 & -s^3 & s^3 \\ 0 & 1 & -1 \\ 0 & 0 & \dfrac{1}{s} \end{pmatrix}$$

The following changes must be made

$$R_{1,1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \; R_{2,1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \; R_{2,4} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$R_2(s) = \begin{pmatrix} s & -s^4 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } p_{1,0} = 2$$

References

[1] Fragulis G., Mertzios B.G. and Vardulakis A.I.G., Computation of the inverse of a polynomial matrix and evaluation of its Laurent expansion., *Int. J. Control*, **53**, pp.431-443.