CIRCUITS SYSTEMS SIGNAL PROCESSING
VOL. 16, NO. 5, 1997, PP. 559–568

COMMENTS ON "REACHABILITY OF POLYNOMIAL MATRIX DESCRIPTIONS (PMDs)" BY G. F. FRAGULIS AND A. I. G. VARDULAKIS*

N. P. Karampetakis[1]

Abstract. The main purpose of these comments is to correct (a) the definitions of admissible initial conditions, (b) the exact solution of a polynomial matrix description (PMD), and (c) the proof of the controllability and reachability criteria, presented in the above-mentioned work. The results concerning the reachability and controllability subspaces and thus the reachability and controllability criteria remain the same.

1. Comments

Consider the homogeneous matrix differential equation

$$A(\rho)\beta(t) = 0 \tag{1}$$

with $A(\rho) = A_0 + A_1\rho + \cdots + A_{q_1}\rho^{q_1} \in \mathbb{R}[\rho]^{r\times r}$ with $\text{rank}_{\mathbb{R}} A(\rho) = r$ and solution according to [2, (3.2)] defined by

$$\beta(t) = (C \quad C_\infty)\begin{bmatrix} e^{Jt}x_s(0-) \\ -\sum_{i=1}^{\hat{q}_r}\delta^{(i-1)}(t)J_\infty^i x_f(0-) \end{bmatrix} \tag{2}$$

We remind that the set of *admissible initial conditions* (AICs) of (1) is the set of states at $t = 0-$ that do not result in impulsive behavior at $t = 0$. Thus we need the term of (2)

$$C_\infty \sum_{i=1}^{\hat{q}_r}\delta^{(i-1)}(t)J_\infty^i x_f(0-) = 0 \tag{3}$$

or equivalently any coefficient of the linearly independent functions $\delta^{(i)}(t)$ for $i = 1, 2, \ldots, \hat{q}_r$ to be zero, i.e.,

$$C_\infty J_\infty x_f(0-) = 0 \;\&\; C_\infty J_\infty^2 x_f(0-) = 0 \;\&\cdots\&\; C_\infty J_\infty^{\hat{q}_r} x_f(0-) = 0 \tag{4}$$

* Received April 12, 1996; accepted April 14, 1997.

[1] Department of Mathematics, Aristotle University of Thessaloniki, Thessaloniki 54006, Greece.

or equivalently

$$x_f(0-) \in \ker \begin{bmatrix} C_\infty J_\infty \\ C_\infty J_\infty^2 \\ \vdots \\ C_\infty J_\infty^{\hat{q}_r} \end{bmatrix} \tag{5}$$

Thus the AICs of (1) are given by

$$H_f = \left\{ x_0 = \begin{pmatrix} x_s(0-) \\ x_f(0-) \end{pmatrix} / [x_s(0-) \neq 0 \text{ s.t. } \beta^{(i)}(0-) = CJ^i x_s(0-), \right.$$
$$\left. i = 0, 1, \dots, q_1], \left[x_f(0-) \in \ker \begin{pmatrix} C_\infty J_\infty \\ C_\infty J_\infty^2 \\ \vdots \\ C_\infty J_\infty^{\hat{q}_r} \end{pmatrix} \right] \right\} \tag{6}$$

instead of $x_f(0-) \in \ker[J_\infty]$ as presented in [2]. It is easily seen in [3], [4] that H_f can be expressed as follows:

$$H_f := \left\{ \beta^{(i)}(0-)\ (i = 0, 1, \dots, q_1 - 1) : \sum_{i=k}^{\hat{q}_r - 1} \sum_{j=0}^{i+k-1} [H_i A_j \beta^{(i+j-k)}(0-)] = 0 \right.$$
$$\left. \text{with } k = 1, 2, \dots, \hat{q}_r \right\} \tag{7}$$

and $\{H_k\}$ is the forward fundamental sequence of $A(s)^{-1}$, i.e.,

$$A(s)^{-1} = H_{\hat{q}_r} s^{\hat{q}_r} + \cdots + H_1 s + H_0 + H_{-1} s^{-1} + \cdots \tag{8}$$

which can be easily computed [1]. Now consider the nonhomogeneous matrix differential equation

$$A(\rho)\beta(t) = B(\rho)u(t) \tag{9}$$

where $B(\rho) = B_0 + B_1\rho + \cdots + B_\sigma \rho^\sigma \in \mathbb{R}[\rho]^{n\times m}$. Taking Laplace transforms in (9) we obtain

$$A(s)\hat{\beta}(s) - \hat{a}(s) = B(s)\hat{u}(s) - \hat{b}(s) \tag{10}$$

where $\hat{\beta}(s) = \mathcal{L}[\beta(t)]$, $\hat{u}(s) = \mathcal{L}[u(t)]$, and

$$\hat{a}(s) = [s^{q-1} I_r, s^{q-2} I_r, \dots, I_r] \begin{bmatrix} A_q & 0 & \cdots & 0 \\ A_{q-1} & A_q & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_1 & A_2 & \cdots & A_q \end{bmatrix} \begin{bmatrix} \beta(0-) \\ \beta^{(1)}(0-) \\ \vdots \\ \beta^{(q_1-1)}(0-) \end{bmatrix}$$
$$\hat{b}(s) = [s^{\sigma-1} I_r, s^{\sigma-2} I_r, \dots, I_r] \begin{bmatrix} B_\sigma & 0 & \cdots & 0 \\ B_{\sigma-1} & B_\sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ B_1 & B_2 & \cdots & B_\sigma \end{bmatrix} \begin{bmatrix} u(0-) \\ u^{(1)}(0-) \\ \vdots \\ u^{(\sigma-1)}(0-) \end{bmatrix} \tag{11}$$

From (10) we have that

$$\hat{\beta}(s) = A(s)^{-1}\hat{a}(s) + A(s)^{-1}B(s)\hat{u}(s) - A(s)^{-1}\hat{b}(s) \tag{12}$$

The inverse Laplace transforms from the first two terms on the right side of (12) have been determined in [2, (2.21) and (2.26)]. However the inverse Laplace transform of $A(s)^{-1}\hat{b}(s)$ has not been determined anywhere and thus is missing from the complete solution presented in [2, (3.12)]. After some manipulations given in [3], [4], we finally obtain

$$\begin{aligned} \mathcal{L}^{-1}\{A(s)^{-1}\hat{b}(s)\} &= Ce^{Jt}u_s(0-) \\ &+ \sum_{i=0}^{\hat{q}_r+\sigma-1}\left\{\sum_{j=0}^{\sigma-1}\sum_{k=\sigma-j}^{\sigma}[H_{-\sigma+1-j+i}B_{\sigma-j}u^{(k-\sigma+j)}(0-)]\right\}\delta^{(i)}(t) \end{aligned} \tag{13}$$

where

$$u_s(0-) = [J^{\sigma-1}B, J^{\sigma-2}B, \dots, B]\begin{bmatrix} B_\sigma & 0 & \cdots & 0 \\ B_{\sigma-1} & B_\sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ B_1 & B_2 & \cdots & B_\sigma \end{bmatrix}\begin{bmatrix} u(0-) \\ u^{(1)}(0-) \\ \vdots \\ u^{(\sigma-1)}(0-) \end{bmatrix} \tag{14}$$

Combining relations (3.12) of [2] and (13), we finally receive according to [3], [4] the complete solution of (9).

For $q_1 < \hat{q}_r$:

$$\begin{aligned} \beta(t) &= -[\delta^{(\hat{q}_r-1)}(t)I_r, \delta^{(\hat{q}_r-2)}(t)I_r, \dots, \delta(t)I_r]\begin{bmatrix} H_{\hat{q}_r} & 0 & & \\ H_{\hat{q}_r-1} & H_{\hat{q}_r} & 0 & \\ \vdots & \vdots & \ddots & \\ H_{\sigma-\hat{q}_r} & H_{\sigma-\hat{q}_r+1} & \cdots & H_{\hat{q}_r} \\ \vdots & \vdots & & \vdots \\ H_1 & H_2 & & H_{q_1} \end{bmatrix} \\ &\times \begin{bmatrix} A_0 & A_1 & \cdots & A_{q_1-1} \\ 0 & A_0 & \cdots & A_{q_1-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_0 \end{bmatrix}\begin{bmatrix} \beta(0-) \\ \beta^{(1)}(0-) \\ \vdots \\ \beta^{(q_1-1)}(0-) \end{bmatrix} \\ &+ Ce^{Jt}[x_s(0-) - u_s(0-)] + \int_0^t Ce^{J(t-\tau)}\Omega u(\tau)\,d\tau \\ &+ [H_{-\sigma}H_{-\sigma+1}\cdots H_0\cdots H_{\hat{q}_r}] \end{aligned}$$

$$\times \begin{bmatrix} B_\sigma & 0 & & & & & \\ B_{\sigma-1} & B_\sigma & & 0 & & & \\ \vdots & \vdots & & & & & \\ B_1 & B_2 & \cdots & & & & \\ B_0 & B_1 & & & & & \\ 0 & B_0 & & & & & \\ \vdots & \vdots & \ddots & & & & \\ 0 & 0 & \cdots & B_0\, B_1 & \cdots & B_{\sigma-1} & B_\sigma \end{bmatrix} \begin{bmatrix} u(t) \\ u^{(1)}(t) \\ \vdots \\ u^{(\sigma-1)}(t) \\ \vdots \\ \vdots \\ u^{(\hat{q}_r+\sigma)}(t) \end{bmatrix}$$

$$+ \left[\delta^{(\hat{q}_r-1)}(t)I_r, \delta^{(\hat{q}_r-2)}(t)I_r, \ldots, \delta(t)I_r\right] \begin{bmatrix} H_{\hat{q}_r} & 0 & \cdots & 0 \\ H_{\hat{q}_r-1} & H_{\hat{q}_r} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_1 & H_2 & \cdots & H_{\hat{q}_r} \end{bmatrix}$$

$$\times \begin{bmatrix} B_0 & B_1 & & \cdots & B_\sigma & & \\ 0 & B_0 & & & & & \\ \vdots & \vdots & \ddots & & & & \\ 0 & 0 & \cdots & B_0\, B_1 & \cdots & B_{\sigma-1} & B_\sigma \end{bmatrix} \begin{bmatrix} u(0-) \\ u^{(1)}(0-) \\ \vdots \\ u^{(\sigma-1)}(0-) \\ \vdots \\ \vdots \\ u^{(\hat{q}_r+\sigma-1)}(0-) \end{bmatrix} \qquad (15a)$$

For $q_1 > \hat{q}_r$:

$$\beta(t) = -\left[\delta^{(\hat{q}_r-1)}(t)I_r, \delta^{(\hat{q}_r-2)}(t)I_r, \ldots, \delta(t)I_r\right] \times \begin{bmatrix} H_{\hat{q}_r} & 0 & \cdots & 0 \\ H_{\hat{q}_r-1} & H_{\hat{q}_r} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_1 & H_2 & \cdots & H_{\hat{q}_r} \end{bmatrix}$$

$$\times \begin{bmatrix} A_0 & A_1 & \cdots & A_{\hat{q}_r-1} & \cdots & A_{q_1-1} \\ 0 & A_0 & \cdots & A_{\hat{q}_r-2} & \cdots & A_{q_1-2} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \cdots \\ 0 & 0 & \cdots & A_0 & \cdots & A_{q_1-\hat{q}_r} \end{bmatrix} \begin{bmatrix} \beta(0-) \\ \beta^{(1)}(0-) \\ \vdots \\ \beta^{(q_1-1)}(0-) \end{bmatrix}$$

$$+ \; Ce^{Jt}[x_s(0-) - u_s(0-)] + \int_0^t Ce^{J(t-\tau)}\Omega u(\tau)\, d\tau$$

$$+ \; [H_{-\sigma}\, H_{-\sigma+1} \cdots H_0 \cdots H_{\hat{q}_r}]$$

$$\times \begin{bmatrix} B_\sigma & 0 & & & & & \\ B_{\sigma-1} & B_\sigma & & & 0 & & \\ \vdots & \vdots & & & & & \\ B_1 & B_2 & \cdots & & & & \\ B_0 & B_1 & & & & & \\ 0 & B_0 & & & & & \\ \vdots & \vdots & \ddots & & & & \\ 0 & 0 & \cdots & B_0\ B_1 & \cdots & B_{\sigma-1} & B_\sigma \end{bmatrix} \begin{bmatrix} u(t) \\ u^{(1)}(t) \\ \vdots \\ u^{(\sigma-1)}(t) \\ \vdots \\ \vdots \\ u^{(\hat{q}_r+\sigma)}(t) \end{bmatrix}$$

$$+ \ [\delta^{(\hat{q}_r-1)}(t)I_r, \delta^{(\hat{q}_r-2)}(t)I_r, \ldots, \delta(t)I_r] \begin{bmatrix} H_{\hat{q}_r} & 0 & \cdots & 0 \\ H_{\hat{q}_r-1} & H_{\hat{q}_r} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_1 & H_2 & \cdots & H_{\hat{q}_r} \end{bmatrix}$$

$$\times \begin{bmatrix} B_0 & B_1 & \cdots & & B_\sigma & & \\ 0 & B_0 & \ddots & & & & \\ \vdots & \vdots & \ddots & & & & \\ 0 & 0 & \cdots & B_0\ B_1 & \cdots & B_{\sigma-1} & B_\sigma \end{bmatrix} \begin{bmatrix} u(0-) \\ u^{(1)}(0-) \\ \vdots \\ u^{(\sigma-1)}(0-) \\ \vdots \\ \vdots \\ u^{(\hat{q}_r+\sigma-1)}(0-) \end{bmatrix} \qquad \text{(15b)}$$

where (i) means the ith (ordinary) derivative of $u(t)$. Following now Definition 3.5 in [2] we can define the admissible initial condition space H_{iu}.

Definition 1. Consider the following space of initial conditions:

$$H_{iu} := \left\{ \begin{bmatrix} \beta(0-) \\ \beta^{(1)}(0-) \\ \vdots \\ \beta^{(q_1-1)}(0-) \end{bmatrix}; \begin{bmatrix} u(0-) \\ u^{(1)}(0-) \\ \vdots \\ u^{(\sigma-1)}(0-) \\ \vdots \\ u^{(\hat{q}_r+\sigma-1)}(0-) \end{bmatrix} : \right.$$

If $q_1 < \hat{q}_r$

$$\begin{bmatrix} H_{\hat{q}_r} & 0 & \cdots & 0 \\ H_{\hat{q}_r-1} & H_{\hat{q}_r} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_1 & H_2 & \cdots & H_{\hat{q}_r} \end{bmatrix}$$

$$
\times \begin{bmatrix} B_0 & B_1 & \cdots & & & B_\sigma & & \\ 0 & B_0 & \ddots & & & & & \\ \vdots & \vdots & \ddots & & & & & \\ 0 & 0 & \cdots & B_0 & B_1 & \cdots & B_{\sigma-1} & B_\sigma \end{bmatrix} \begin{bmatrix} u(0-) \\ u^{(1)}(0-) \\ \vdots \\ u^{(\sigma-1)}(0-) \\ \vdots \\ \vdots \\ u^{(\hat{q}_r+\sigma-1)}(0-) \end{bmatrix}
$$

$$
- \begin{bmatrix} H_{\hat{q}_r} & 0 & & \\ H_{\hat{q}_r-1} & H_{\hat{q}_r} & 0 & \\ \vdots & \vdots & \ddots & \\ H_{\sigma-\hat{q}_r} & H_{\sigma-\hat{q}_r+1} & \cdots & H_{\hat{q}_r} \\ \vdots & \vdots & & \vdots \\ H_1 & H_2 & & H_{q_1} \end{bmatrix}
$$

$$
\cdot \begin{bmatrix} A_0 & A_1 & \cdots & A_{q_1-1} \\ 0 & A_0 & \cdots & A_{q_1-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_0 \end{bmatrix} \begin{bmatrix} \beta(0-) \\ \beta^{(1)}(0-) \\ \vdots \\ \beta^{(q_1-1)}(0-) \end{bmatrix} = 0 \tag{16a}
$$

If $q_1 > \hat{q}_r$

$$
\begin{bmatrix} H_{\hat{q}_r} & 0 & \cdots & 0 \\ H_{\hat{q}_r-1} & H_{\hat{q}_r} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_1 & H_2 & \cdots & H_{\hat{q}_r} \end{bmatrix}
$$

$$
\times \begin{bmatrix} B_0 & B_1 & \cdots & & & B_\sigma & & \\ 0 & B_0 & \ddots & & & & & \\ \vdots & \vdots & \ddots & & & & & \\ 0 & 0 & \cdots & B_0 & B_1 & \cdots & B_{\sigma-1} & B_\sigma \end{bmatrix} \begin{bmatrix} u(0-) \\ u^{(1)}(0-) \\ \vdots \\ u^{(\sigma-1)}(0-) \\ \vdots \\ \vdots \\ u^{(\hat{q}_r+\sigma-1)}(0-) \end{bmatrix}
$$

$$
\begin{bmatrix} H_{\hat{q}_r} & 0 & \cdots & 0 \\ H_{\hat{q}_r-1} & H_{\hat{q}_r} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_1 & H_2 & \cdots & H_{\hat{q}_r} \end{bmatrix}
$$

$$
\left.\cdot \begin{bmatrix} A_0 & A_1 & \cdots & A_{\hat{q}_r-1} & \cdots & A_{q_1-1} \\ 0 & A_0 & \cdots & A_{\hat{q}_r-2} & \cdots & A_{q_1-2} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_0 & \cdots & A_{q_1-\hat{q}_r} \end{bmatrix} \begin{bmatrix} \beta(0-) \\ \beta^{(1)}(0-) \\ \vdots \\ \beta^{(q_1-1)}(0-) \end{bmatrix} = 0 \right\} \tag{16b}
$$

or, equivalently,

$$H_{iu} := \Bigg\{ \beta^{(i)}(0-)\ i = 0, 1, \ldots, q_1 - 1,\ u^{(i)}(0-)\ i = 0, 1, \ldots, \hat{q}_r + \sigma - 1 : \\ \sum_{i=k}^{\hat{q}_r} H_i \left[\sum_{j=0}^{\sigma} B_j u^{(i+j-k)}(0-) - \sum_{j=0}^{q_1-i+k-1} A_j y^{(i+j-k)}(0-) \right] = 0, \\ k = 1, 2, \ldots, \hat{q}_r \Bigg\} \tag{17}$$

H_{iu} will be called the *admissible initial condition space* of the ARMA representation (9) for every $u(t)$.

It is easily seen from (15) that under the preceding set of initial conditions, the solution (19) of the ARMA representation (9) will be free of impulses. This is the reason we call these conditions admissible. Under the preceding initial conditions and not the ones presented in [2] the solution of the ARMA representation (9) will be the following:

$$\beta(t) = Ce^{Jt}[x_s(0-) - u_s(0-)] + \int_0^t Ce^{J(t-\tau)}\Omega u(\tau)\, d\tau \\ + [H_{-\sigma} H_{-\sigma+1} \cdots H_0 \cdots H_{\hat{q}_r}] \\ \times \begin{bmatrix} B_\sigma & 0 & & & & \\ B_{\sigma-1} & B_\sigma & & 0 & & \\ \vdots & \vdots & \ddots & & & \\ B_1 & B_2 & \cdots & & & \\ B_0 & B_1 & & & & \\ 0 & B_0 & & & & \\ \vdots & \vdots & \ddots & & & \\ 0 & 0 & \cdots & B_0\ B_1 & \cdots & B_{\sigma-1}\ B_\sigma \end{bmatrix} \begin{bmatrix} u(t) \\ u^{(1)}(t) \\ \vdots \\ u^{(\sigma-1)}(t) \\ \vdots \\ \vdots \\ u^{(\hat{q}_r+\sigma)}(t) \end{bmatrix} \tag{18}$$

or, equivalently,

$$\beta(t) = Ce^{Jt}[x_s(0-) - u_s(0-)] + \int_0^t Ce^{J(t-\tau)}\Omega u(\tau)\, d\tau \\ + \sum_{i=0}^{\sigma} \sum_{j=i}^{\sigma} H_{-i} B_j u^{(k+j-1)}(t) + \sum_{i=1}^{\hat{q}_r} \sum_{j=0}^{\sigma} H_i B_j u^{(j+i)}(t) \tag{19}$$

The only difference between the solution (19) and the one presented in [2, (3.29)] is the extra term $-Ce^{Jt}u_s(0-)$ in (19).

In the case where the initial conditions of the inputs at $t = 0-$ are zero, i.e., $u^{(i)}(0-) = 0$, $i = 0, 1, \ldots, \sigma - 1$, the smooth solutions presented in (19) and [2, (3.29)] are the same because the term $Ce^{Jt}u_s(0-)$ vanishes.

In the fourth section and more specifically in the proof of Theorem 4.6, the authors say that $\beta^c(0-) = 0$ implies $x_s(0-) \equiv 0$, $u^{(i)}(0-) \equiv 0$, and $\beta^c(t)$ is impulse free for $t > 0$. They use this result for the remaining proof of Theorem 4.6. Obviously this is not true, as we can see from what follows.

$$
\begin{aligned}
&\beta^c(0-) = 0 \overset{(19)}{\Leftrightarrow} Ce^{J0}[x_s(0-) - u_s(0-)] + \int_0^{0-} Ce^{J(t-\tau)}\Omega u(\tau)\,d\tau \\
&+ \sum_{i=0}^{\sigma}\sum_{j=i}^{\sigma} H_{-i}B_j u^{(k+j-1)}(0-) + \sum_{i=1}^{\hat{q}_r}\sum_{j=0}^{\sigma} H_i B_j u^{(j+i)}(0-) = 0 \\
&\Leftrightarrow C[x_s(0-) - u_s(0-)] + \sum_{i=0}^{\sigma}\sum_{j=i}^{\sigma} H_{-i}B_j u^{(k+j-1)}(0-) \\
&+ \sum_{i=1}^{\hat{q}_r}\sum_{j=0}^{\sigma} H_i B_j u^{(j+i)}(0-) = 0 \\
&\Leftrightarrow \sum_{i=0}^{q-1}\sum_{j=i+1}^{q} H_{-j+i}A_j\beta^{(i)}(0-) - \sum_{i=0}^{\sigma-1}\sum_{j=i+1}^{\sigma} H_{-j+i}B_j u^{(i)}(0-) \\
&+ \sum_{i=0}^{\sigma}\sum_{j=i}^{\sigma} H_{-i}B_j u^{(k+j-1)}(0-) + \sum_{i=1}^{\hat{q}_r}\sum_{j=0}^{\sigma} H_i B_j u^{(j+i)}(0-) = 0 \\
&\Leftrightarrow \sum_{i=0}^{q-1}\sum_{j=i+1}^{q} H_{-j+i}A_j\beta^{(i)}(0-) + \sum_{i=0}^{\hat{q}_r}\sum_{j=0}^{\sigma} H_i B_j u^{(j+i)}(0-) = 0. \qquad (20)
\end{aligned}
$$

We can easily observe using (14), (19), and [2, (4.13)], that

$$
\beta^c(T) - Ce^{JT}x_s(0-) \in \operatorname{Im}[H_{-\sigma}H_{-\sigma+1}\cdots H_0\cdots H_{\hat{q}_r}]
\times \begin{bmatrix}
B_\sigma & 0 & & & & & \\
B_{\sigma-1} & B_\sigma & & & 0 & & \\
\vdots & \vdots & \ddots & & & & \\
B_1 & B_2 & \cdots & & & & \\
B_0 & B_1 & & & & & \\
0 & B_0 & & & & & \\
\vdots & \vdots & \ddots & & & & \\
0 & 0 & \cdots & B_0\ B_1 & \cdots & B_{\sigma-1} & B_\sigma
\end{bmatrix}
+ C\langle J/\operatorname{Im}\Omega\rangle \qquad (21)
$$

or, equivalently, from [2, (2.27) and (4.2)] that

$$
\beta^c(T) - Ce^{JT}x_s(0-) \in \operatorname{Im}[H_{-\sigma-n}H_{-\sigma-n-1}\cdots H_0\cdots H_{\hat{q}_r}]
$$

$$\times \begin{bmatrix} B_\sigma & 0 & & & & & \\ B_{\sigma-1} & B_\sigma & & & 0 & & \\ \vdots & \vdots & \ddots & & & & \\ B_1 & B_2 & \cdots & & & & \\ B_0 & B_1 & & & & & \\ 0 & B_0 & & & & & \\ \vdots & \vdots & \ddots & & & & \\ 0 & 0 & \cdots & B_0\ B_1 & \cdots & B_{\sigma-1} & B_\sigma \end{bmatrix} \tag{22}$$

where $n := \deg[A(s)]$. From (22) it is easily seen that a necessary condition for (9) to be reachable is that

$$\mathrm{Rank}_{\mathbb{R}}[H_{-\sigma-n}\, H_{-\sigma-n-1} \cdots H_0 \cdots H_{\hat{q}_r}] \times \begin{bmatrix} B_\sigma & 0 & & & & & \\ B_{\sigma-1} & B_\sigma & & & 0 & & \\ \vdots & \vdots & \ddots & & & & \\ B_1 & B_2 & \cdots & & & & \\ B_0 & B_1 & & & & & \\ 0 & B_0 & & & & & \\ \vdots & \vdots & \ddots & & & & \\ 0 & 0 & \cdots & B_0\ B_1 & \cdots & B_{\sigma-1} & B_\sigma \end{bmatrix} = r \tag{23}$$

The compound matrix in (23) coincides with the reachability matrix presented in [2, (4.62)]. However the criteria test in (23) depends on the fundamental sequence matrices $\{H_i\}$ of the inverse of $A(s)$, which are easily computed [1], instead of the finite and infinite Jordan pairs whose computation is numerically unstable. The sufficiency of the preceding reachability condition may come from the second part of the proof of Theorem 4.6 in [2] with slight changes.

2. Conclusion

The purpose of this work is to correct the following points in [2]: (a) the definitions of admissible initial conditions for homogeneous and nonhomogeneous matrix differential equations, (b) the solution of nonhomogeneous matrix differential equations, and (c) the proof of Theorem 4.6 concerning the reachable subspace of nonhomogeneous matrix differential equations. Some slight changes to the proof of the results in Section 4 (Theorem 4.6) will give rise to the same conclusions. However, one can refer to [5] for further results concerning the reachability, controllability, and observability of polynomial matrix descriptions (PMDs).

References

[1] Fraglis, G.; Mertzios, B. G.; and Vardulakis, A. I., Computation of the inverse of a polynomial matrix and evaluation of its Laurent expansion, *Int. J. Control*, **53**, 431–443, 1991.

[2] Fragulis, G., and Vardulakis, A. I., Reachability of polynomial matrix descriptions (PMDs), *Circuit Systems Signal Process*, **14**, 787–815, 1995.

[3] Karampetakis, N. P., and Vardulakis, A. I., On the solution of ARMA representations, *Proceedings of the Third IEEE Mediterranean Symposium on New Directions in Control and Automation*, pp. 156–163, 1995.

[4] Karampetakis, N. P., and Vardulakis, A. I., A closed formula for the solution of an ARMA representation, *Mathematical Report No. A241*, Department of Mathematics, Loughborough University of Technology, Loughborough, U.K., 1995.

[5] Mahmood, S.; Karampetakis, N. P.; and Pugh, A. C. Structural properties of regular PMDs, to appear in *Int. J. Control*.