Solution of Discrete ARMA-Representations via MAPLE

by

J. Jones†, N. P. Karampetakis‡ and A.C. Pugh†

† Department of Mathematics
Loughborough University of Technology
Loughborough, Leics LE11 3TU
England, U.K.

‡ Department of Mathematics
Aristotle University of Thessaloniki
Thessaloniki 54006, Greece

Abstract.

In Karampetakis et. al. (1996) closed formulae for the forward, backward and symmetric solutions of an ARMA-Representation have been presented. Here these formulae are implemented in the symbolic computational language MAPLE and corresponding MAPLE code is provided.

PROBLEM STATEMENT

Let the ARMA-Representation

$$A(\sigma)y(k) = B(\sigma)u(k) \qquad (1)$$

where

$$A(\sigma) = A_0 + A_1\sigma + \ldots + A_q\sigma^q \in R[\sigma]^{r\times r}, \ \ \text{rank}_{R(\sigma)}A(\sigma) = r$$

$$B(\sigma) = B_0 + B_1\sigma + \ldots + B_q\sigma^q \in R[\sigma]^{r\times m}$$

where at least one of A_q, B_q is nonzero, σ denotes the advance operator (i.e. $\sigma^i y(k) = y(k+i)$), $y(k): Z^+ \to R^r$ defines the *output* and $u(k): Z^+ \to R^m$ defines the *input* of the system (1).

We are interested to determine via the symbolic computational package MAPLE the following three solutions of the equation (1) :

1. **FORWARD** - Given a set of admissible initial conditions {y(0),y(1), ..., y(q-1)} determine y(k), (k=q,q+1,...) in a *forward* fashion from the input sequence u(k) and previous q values of the output {y(k-1),y(k-2),...,y(k-q)}.

2. **BACKWARD** - Given a set of admissible final conditions {y(N),y(N-1),...,y(N-q+1)} determine y(k) (k=N-q,N-q-1,...) in a *backwars* fashion from the input sequence u(k) and the future q values of the output {y(k+1),y(k+2),...,y(k+q)}.

3. **SYMMETRIC** - Given a set of admissible initial and final conditions, {y(0),y(1),...,y(q-1)} and {y(N),y(N-1),...,y(N-q+1)}, determine y(k) for $k \in [q, N-q]$ from the input sequence u(k) and the q previous (forward symmetric) or q future outputs (backward symmetric).

SYMBOLIC COMPUTATIONAL PACKAGES ?

- Computer enviroments for doing Mathematics.
- Particularly applicable to "computationally attractive" problems.
- Works with symbolic, numerical and graphical computations.

ADVANTAGES OF SYMBOLIC COMPUTATIONAL PACKAGES

1. **Symbolic storage.**

   - Variables can be stored in an **"exact"** form (i.e. 1/3 as opposed to 0.33333..) resulting in no loss of accuracy during a calculation.

   - Variables can be left **"unassigned"** (i.e. without holding any numerical values) which enables polynomial operations to be defined, say, in an indeterminate s.

2. **Inbuilt Procedures.**

   The existence of hundrends of inbuilt procedures covering both general and certain specialized areas of mathematics.

3. **Programming Language.**

   The existence of unique high-level programming languages allowing specific procedures to be written.

DISADVANTAGES OF SYMBOLIC COMPUTATIONAL PACKAGES

1. Large size of memory they use,
2. Slow speed they have,

when they use numbers in "exact" form or "unassigned" variables.

IS MAPLE APPLICABLE TO LINEAR SYSTEMS ? YES!

- The linear algebra package *linalg*

  - Consists of numerous procedures used in the solution of problems in linear algebra.
    Examples include
    i) *inverse* - Computes the inverse of a nonsingular matrix.
    ii) *linsolve* - Solves a set of linear equations.

  - Can be viewed upon as building blocks for subsequently more specified and advanced procedures.

- The ability to perform "polynomial operations".

- Facility exists to create additional packages.

MOTIVATION AND AIM

- No package exists for specialized work in solutions of discrete time ARMA-representations.

- Symbolic packages are flexible enough for the implementation of such work to be carried out.

- Develop a package of procedures for solving numerous problems in linear systems via Maple.

SYSTEM SOLUTIONS PACKAGE

Maple name - *linsol*

- Contains procedures for obtaining forward, backward and symmetric solutions of the system (1).

- Comprises of 22 procedures.

Procedures - [ARSOLVE, BACK, BACKADMIS, BACKSYM, COMSOLVE, DIFFPOW, FORADMIS, FORSYM, FORWARD, GINVERSE, IDEN, MATCH, MATEXP, PLINSOLVE, PLOTSOL, SOLBACK, SOLBACKSYM, SOLFOR, SOLFORSYM, SOLSYM, SYMADMISS, SYMMETRIC]

PACKAGE IMPLEMENTATION

- Load the Maple package *linalg* via
  with(linalg):

- Load the package *linsol* into maple via
  > with(linsol):

- Procedures can be directly implemented via
  > file1(args1);

- The package has been implemented on a SUN SPARC station 10 (75Mhz SuperSPARC II).

MAPLE PROCEDURES

1. Forward ARMA-Representation solution.

Maple Procedure - FORWARD(A,B,q)

Input : The polynomial matrices A(σ), B(σ) which define the ARMA-Representation in (1). q is the greatest degree of all the polynomial entries in A(σ), B(σ).

Output : The *forward* solution y(k) of (1) in terms of the q previous outputs {y(k-1), y(k-2), ..., y(k-q)} and the input sequence u(k) (see Karampetakis et. al. (1996)).

Maple Procedure - FORADMIS(A,B,q)

Input : The polynomial matrices A(σ), B(σ) which define the ARMA-Representation in (1). q is the greatest degree of all the polynomial entries in A(σ), B(σ).

Output : The admissible initial conditions {y(0),y(1),...,y(q-1)} of (1) according to Karampetakis et. al. (1996).

2. Backward ARMA-Representation solution.

Maple Procedure - BACK(A,B,q)

Input : The polynomial matrices A(σ), B(σ) which define the ARMA-Representation in (1). q is the greatest degree of all the polynomial entries in A(σ), B(σ).

Output : The *backward* solution y(k) of the regular, discrete-time ARMA-Representation (1) in terms of the q future outputs {y(k+1), y(k+2), ..., y(k+q)} and the input sequence u(k) (see Karampetakis et. al. (1996)).

Maple Procedure - BACKADMIS(A,B,q)

Input : The polynomial matrices A(σ), B(σ) which define the ARMA-Representation in (1). q is the greatest degree of all the polynomial entries in A(σ), B(σ).

Output : The admissible *final* conditions {y(N),y(N-1),...,y(N-q+1)} of (1) (see Karampetakis et. al. (1996)).

3. Symmetric ARMA-Representation solution.

Maple Procedure - FORSYM(A,B,q,N,k)

Input : The polynomial matrices A(σ), B(σ) which define the ARMA-Representation in (1). q is the greatest degree of all the polynomial entries in A(σ), B(σ). [0, N] is the discrete-time domain. y(k) is the output we are looking for.

Output : The *forward-symmetric* solution of (1) in terms of the q previous outputs {y(k-1),y(k-2),...,y(k-q)}, the final conditions {y(N),y(N-1),..,y(N-q+1)} and the input sequence u(k) (see Karampetakis et. al. (1996)).

Maple Procedure - BACKSYM(A,B,q,N,k)

Input : The polynomial matrices A(σ), B(σ) which define the ARMA-Representation in (1). q is the greatest degree of all the polynomial entries in A(σ), B(σ). [0, N] is the discrete-time domain. y(k) is the output we are looking for.

Output : The *backward-symmetric* solution of (1) in terms of the q future outputs {y(k+1),y(k+2),...,y(k+q)}, the initial conditions {y(0),y(1),..,y(q-1)} and the input sequence u(k) (see Karampetakis et. al. (1996)).

ILLUSTRATED EXAMPLE

```
> with(linalg):
  Warning: new definition for   norm
  Warning: new definition for   trace

> with(linsol):

> A:=matrix(3,3,[s^2+5*s,s+1,0,2*s,3*s,1,0,-1,0]);
```

$$A := \begin{bmatrix} s^2+5\,s & s+1 & 0 \\ 2\,s & 3\,s & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

```
> B:=matrix(3,1,[0,0,1]);
```

$$B := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

```
> FORWARD(A,B,2);
```

$$\begin{bmatrix} -5\,ykmin11 - 5\,ykmin22 - 4\,ukmin21 + ukmin11 \\ -ukmin01 \\ -50\,ykmin11 - 50\,ykmin22 - 40\,ukmin21 + 8\,ukmin11 - 2\,ukmin01 + 3\,ukplus11 \end{bmatrix}$$

```
> FORADMIS(A,B,2);
```

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & -3 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & -4 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}, \begin{bmatrix} ykplus01 \\ ykplus02 \\ ykplus03 \\ ykplus11 \\ ykplus12 \\ ykplus13 \end{bmatrix}, =, \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 5 & -2 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} ukplus01 \\ ukplus11 \\ ukplus21 \\ ukplus31 \\ ukplus41 \end{bmatrix}$$

```
> BACK(A,B,2);
```

$$\begin{bmatrix} \frac{1}{25}\,ykplus21 + \frac{1}{25}\,ykplus12 + \frac{4}{25}\,ukplus01 + \frac{1}{5}\,ukmin11 \\ -ukplus01 \\ \frac{2}{5}\,ykplus21 - \frac{13}{5}\,ykplus12 - \frac{2}{5}\,ukplus01 \end{bmatrix}$$

```
> BACKADMIS(A,B,2);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} yNmin01 \\ yNmin02 \\ yNmin03 \\ yNmin11 \\ yNmin12 \\ yNmin13 \end{bmatrix}, =, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{5} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} uNmin01 \\ uNmin11 \\ uNmin21 \end{bmatrix}$$

```
> FORSYM(A,B,2,10,3);
```

$$\begin{bmatrix} -5\,y21 - 5\,y12 + u21 - 4\,u11 \\ -u31 \\ -50\,y21 - 50\,y12 + 3\,u41 - 2\,u31 + 8\,u21 - 40\,u11 \end{bmatrix}$$

```
> BACKSYM(A,B,2,10,7);
```

$[-1953125\, y11 - 1953125\, y02 + u91 - 4\, u81 + 20\, u71 - 100\, u61 + 500\, u51 - 2500\, u41$
$+ 12500\, u31 - 62500\, u21 + 312500\, u11 - 1562500\, u01]$

$[-u101]$

$[-19531250\, y11 - 19531250\, y02 - 3\, y112 - 2\, u101 + 8\, u91 - 40\, u81 + 200\, u71$
$- 1000\, u61 + 5000\, u51 - 25000\, u41 + 125000\, u31 - 625000\, u21 + 3125000\, u11$
$- 15625000\, u01]$

NOTE : Here we use the following notation

$$y(k+1) = \begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ y_3(k+1) \end{bmatrix} = \begin{bmatrix} ykplus11 \\ ykplus12 \\ ykplus13 \end{bmatrix}$$

CONCLUSIONS

- Advantages of Maple in solving problems in Linear Systems.
  - linalg package
  - symbolic operations
- Developed and presented a package of procedures for use in Linear Systems.
- The package is easily implemented.
  - self contained
  - illustrated by several numerical examples
- Easily expandable.
  - add to existing package.txt file
  - (alternatively) create new package
- By no means a complete package.
- Clear advantage for use in the area of Linear Systems.
- Applications in Education & Industry.