Structural Properties of Regular PMDs

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Abstract

Consider the generalised state space system (GSSS)

$$E\dot{x}(t) = Ax(t) + \mathcal{B}u(t) \tag{1a}$$
$$y(t) = \mathcal{C}x(t) \tag{1b}$$

where $\rho E_n - A \in \mathbb{R}[\rho]^{l\times l}$, $\mathcal{B} \in \mathbb{R}^{l\times m}$, $\mathcal{C} \in \mathbb{R}^{p\times l}$, $\text{rank}_{\mathbb{R}} E < l$, (in which case $n := \deg|\rho E - A| < l$) and where $x(t) : [0-, +\infty) \to \mathbb{R}^l$ is the state vector, $u(t) : [0-, +\infty) \to \mathbb{R}^m$ is the input vector and $y(t) : [0-, +\infty) \to \mathbb{R}^p$ is the output vector of (1). In case $E \in \mathbb{R}^{l\times l}$ is non-singular then (1) is the known state space representation widely studied by [5], [6], [7].

Assuming that the pencil $sE - A \in \mathbb{R}[s]^{l\times l}$ has at least one zero at $s = \infty$ then (1) may also be written in *standard canonical form* ([3])

$$\dot{x}_1 = E_1 x_1(t) + B_1(t)u(t) \tag{2a}$$
$$E_2\dot{x}_2(t) = x_2(t) + B_2(t)u(t) \tag{2b}$$
$$y(t) = [C_1 \ C_2]\begin{bmatrix} x_1(t) \\ x_2(t)\end{bmatrix} \tag{2c}$$

where $E_1 \in \mathbb{R}^{r_1\times r_1}$, $E_2 \in \mathbb{R}^{r_2\times r_2}(E_2^{\eta} = 0,\ E_2^{\eta-1} \neq 0)$ with $r_1 + r_2 = l$, $B_1 \in \mathbb{R}^{r_1\times m}$, $B_2 \in \mathbb{R}^{r_2\times m}$ and η is the degree of nilpotency of E_2.

Consider (2) with $u(t) : [0,\infty) \to \mathbb{R}^l$ then the complete solution $x_{\text{com}}(t)$, of (2) with non-zero initial values $x^{(i)}(0-)$ is given by ([2])

$$x_{\text{com}}(t) = x_1(t) + x_2(t) \tag{3a}$$

where

$$x_1(t) = e^{E_1 t}x_1(0-) + \int_{0-}^{t} e^{E_1(t-\tau)}B_1 u(\tau)\, \mathrm{d}\tau \tag{3b}$$

$$x_2(t) = \sum_{i=0}^{\eta-1} E_2^i B_2 u^{[i]}(t) + \sum_{i=0}^{\eta-2}\delta^{(i)}(t)E_2^i\Big[(-1)E_2x_2(0-) + \sum_{j=0}^{\eta-2-i} E_2^{j+1}B_2u^{[j]}(0)\Big] \tag{3c}$$

where $u^{[i]}(t)$ denotes the regular derivative of $u(t)$. As can be seen from (3c), the complete solution of the GSSS (2) exhibits impulsive behaviour and in this regard, we call a $x(0-) = [x_1(0-)^T \ x_2(0-)^T]^T \in \mathbb{R}^l$ an *impulse free initial value* for (2) if there exists an input $u(t)$ such that the complete solution $x_{\text{com}}(t)$ is continuously differentiable on $[0,T]$ for some $T > 0$ (i.e., $x_{\text{com}}(t)$ is "impulse free" on $[0,T]$). This then leads to the following.

Definition 1. The space of initial values

$$\mathcal{A}_{iu} := \left\{ x(0-) = \begin{bmatrix} x_1(0-) \\ x_2(0-)\end{bmatrix} : x_1(0-) \in \mathbb{R}^{r_1},\ x_2(0-) \in \ker E_2 + \sum_{i=0}^{\eta-2} E_2^i\ \text{Im}B_2 \in \mathbb{R}^{r_2} \right\}. \tag{4}$$

are called the set of *impulse free initial values* for (2). □

Notice that $x(0-) = 0 \in \mathbb{R}^l$ belongs to $\mathcal{A}_{iu}$ because we can take $x_1(0-) = 0 \in \mathbb{R}^{r_1}$ and $u(t)$ such that $u^{[i]}(0) = 0, i = 0, 1, \ldots, r_2 - 2$. Thus $\mathcal{A}_{iu}$ is a subspace of $\mathbb{R}^l$. With this background we can now define state reachability, controllability and observability for (2).

Definition 2. Given a point $x_0 = x(0-) \in \mathcal{A}_{iu}$, a point $x_T \in \mathbb{R}^l$ is *state reachable* (SR) from x_0 if there exists a $u(t)$ and $T > 0$ such that $x_{\text{com}}(t)$ is continuously differentiable on [0,T] and $x_{\text{com}}(T) = x_T$. If $x_0 = x(0-) \neq 0$ and $x_{\text{com}}(T) = x_T = 0 \in \mathbb{R}^l$, i.e., if the "origin" $0 \in \mathbb{R}^l$ is SR from x_0 then we say that x_0 is *state controllable* (SC). A point $x_T \in \mathbb{R}^l$ is *state observable* (SO) if the evolution of $x(T)$ for $T \geq 0$ may be determined using only knowledge of $u(T)$ and $y(T)$ for $T \geq 0$. □

We define the notation $< \cdot | \cdot >$ for an arbitrary matrix pair (E, B), where E is a square matrix and the product EB is well defined:

$$< E|B > := \text{Im}B + E\text{Im}B + \cdots + E^{j-1}\text{Im}B \tag{5}$$

where j is the order of E and $\text{Im}B = \{x : x = By, \text{ for all possible } y\}$. Let now $\mathbf{R}$ be the complete set of $x_T \in \mathbb{R}^l$ SR from any $x_0 \in \mathcal{A}_{iu}$ and $\tilde{\mathbf{R}}$ be the set of $x_0 \in \mathbb{R}^l$ which can reach $x_T = 0$.

Lemma 1. ([3], [2])

(a) The *SR subspace* of (2) is given by $\mathbf{R} = \{< E_1|B_1 > \oplus < E_2|B_2 >\}$ and thus every $x_T \in \mathbb{R}^l$ is SR iff

$$\text{rank}_{\mathbb{R}} \begin{bmatrix} Q_1 & 0_{r_1, r_2 m} \\ 0_{r_2, r_1 m} & Q_2 \end{bmatrix} \in \mathbb{R}^{(r_1+r_2)\times(r_1+r_2)m} = l, \tag{6}$$

(b) The *SC subspace* of (2) is given by $\tilde{\mathbf{R}} = \{< E_1|B_1 > \oplus [\ < E_2|B_2 > + \ker E_2\ ]\ \}$ and thus every $x_0 \in \mathbb{R}^l$ is SC iff

$$\text{rank}_{\mathbb{R}} \begin{bmatrix} Q_1 & 0_{r_1,(r_2 m+\sigma)} \\ 0_{r_2, r_1 m} & Q_2, N(E_2) \end{bmatrix} \in \mathbb{R}^{(r_1+r_2)\times[m(r_1+r_2)+\sigma]} = l, \tag{7}$$

(c) Every $x_T \in \mathbb{R}^l$ is SO iff

$$\text{rank}_{\mathbb{R}} \begin{bmatrix} C_1 \\ C_1 E_1 \\ \vdots \\ C_1 E_1^{r_1 - 1} \end{bmatrix} = r_1, \tag{8}$$

where $Q_1 := [B_1, E_1 B_1, \ldots, E_1^{r_1-1} B_1] \in \mathbb{R}^{r_1 \times r_1 m}$, $Q_2 := [B_2, E_2 B_2, \ldots, E_2^{r_2-1} B_2] \in \mathbb{R}^{r_2 \times r_2 m}$ are the finite and infinite state reachability matrices respectively of (2) and where $N(E_2) \in \mathbb{R}^{r_2 \times \sigma}$, $\sigma = r_2 - \epsilon$, $\epsilon = \text{rank}_{\mathbb{R}} E_2$, a basis matrix for $\ker E_2$. □

Thus it is evident from the above that state reachability of (2) implies its state controllablity but the converse is not true.

Consider now a linear time invariant multivariable system Σ described by a PMD:

$$A(\rho)\beta(t) = B(\rho)u(t) \tag{9a}$$

$$y(t) = C(\rho)\beta(t) + D(\rho)u(t) \tag{9b}$$

where $(\rho = d/dt), A(\rho) \in \mathbb{R}[\rho]^{l\times l}$ with $|A(\rho)| \neq 0, B(\rho) \in \mathbb{R}[\rho]^{l\times m}, C(\rho) \in \mathbb{R}[\rho]^{p\times l}, D(\rho) \in \mathbb{R}[\rho]^{p\times m}, \beta(t) : [0-, \infty) \to \mathbb{R}^l$ is the *pseudo state* of $\Sigma, u(t) : [0-, \infty) \to \mathbb{R}^m$ is the *control input* and $y(t)$ the *output* of Σ. Σ may be written in the form:

$$\underbrace{\begin{bmatrix} A(\rho) & B(\rho) & 0 \\ -C(\rho) & D(\rho) & I_p \\ 0 & -I_m & 0 \end{bmatrix}}_{T(\rho)} \underbrace{\begin{bmatrix} \beta(t) \\ -u(t) \\ y(t) \end{bmatrix}}_{\xi(t)} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ I_m \end{bmatrix}}_{\mathcal{U}} u(t), \qquad y(t) = \underbrace{\begin{bmatrix} 0 & 0 & I_p \end{bmatrix}}_{\mathcal{V}} \underbrace{\begin{bmatrix} \beta(t) \\ -u(t) \\ y(t) \end{bmatrix}}_{\xi(t)} \tag{10}$$

where $\mathcal{T}(\rho) = \mathcal{T}_0 + \mathcal{T}_1\rho + \cdots + \mathcal{T}_q\rho^q \in \mathbb{R}[\rho]^{r\times r}$ with $r = l + p + m = \text{rank}(\mathcal{T}(\rho))$, $\mathcal{U} \in \mathbb{R}^{r\times m}$, $\mathcal{V} \in \mathbb{R}^{p\times r}$, $\xi(t) : [0-, +\infty) \to \mathbb{R}^r$ is the pseudostate of the *normalised system* (10).

[1] proposed an algorithm which reduces a general PMD in normalised form (10) to a GSSS with the same finite and infinite frequency properties. The equivalent GSSS can be found to be of the form

$$\underbrace{\begin{pmatrix} I_n & 0_{n,\mu} \\ 0_{\mu,n} & -J_\infty \end{pmatrix}}_{E} \underbrace{\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{pmatrix} J & 0_{n,\mu} \\ 0_{\mu,n} & -I_\mu \end{pmatrix}}_{A} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{pmatrix} B\mathcal{U} \\ B_\infty\mathcal{U} \end{pmatrix}}_{\mathcal{B}} u(t) \tag{11b}$$

$$y(t) = \underbrace{\begin{pmatrix} \mathcal{V}C & \mathcal{V}C_\infty \end{pmatrix}}_{\mathcal{C}} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{x(t)} \tag{11b}$$

where $[C \in \mathbb{R}^{r\times n}, J \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times r}]$ is a minimal realisation of $H_{\text{sp}}(s)$ (the strictly proper part of $\mathcal{T}(s)^{-1}$) and $[C_\infty \in \mathbb{R}^{r\times\mu}, J_\infty \in \mathbb{R}^{\mu\times\mu}, B_\infty \in \mathbb{R}^{\mu\times r}]$ is a minimal realisation of $H_{\text{pol}}(s)$ (the polynomial part of $\mathcal{T}(s)^{-1}$), i.e.,

$$\mathcal{T}(s)^{-1} = C(sI_n - J)^{-1}B + C_\infty(I_\mu - sJ_\infty)^{-1}B_\infty. \tag{12}$$

The following Theorem shows that the PMD (10) and the equivalent GSSS (11) are related via bijective maps between their solution/input pairs.

Theorem 1. The maps between the solution/input pairs of the PMD (10) and the GSSS (11)

$$\begin{pmatrix} \xi(t) \\ -u(t) \end{pmatrix} = \begin{pmatrix} (C\ C_\infty) & 0_{r,m} \\ 0_{m,(n+\mu)} & I_m \end{pmatrix} \begin{pmatrix} x(t) \\ -u(t) \end{pmatrix} \tag{13a}$$

$$\begin{pmatrix} x(t) \\ -u(t) \end{pmatrix} = \begin{pmatrix} (\rho E - A)^{-1}) \begin{pmatrix} B \\ B_\infty \end{pmatrix} \mathcal{T}(\rho) & 0_{(n+\mu),m} \\ 0_{m,r} & I_m \end{pmatrix} \begin{pmatrix} \xi(t) \\ -u(t) \end{pmatrix} \tag{13b}$$

are both bijective. □

Equivalence between the GSSS and the PMD basically requires that the solution spaces of two equivalent systems be isomorphic. The initial conditions represent a section of the solution space taken at time $t = 0-$, and thus represent a set of points from which solutions emanate at this time. It can therefore be expected that the bijection which exists between the solution spaces of equivalent systems would induce a bijection between the corresponding sets of initial conditions. Further since the initial conditions sets are merely sets of "points" it could be expected that this bijection would be a constant map (see also [4], [11]). These conjectures are confirmed by the following result.

Theorem 2. There exists bijective maps between the initial conditions $\mathcal{X}_T\hat{\xi}(0-)$, of the PMD (10) and the initial conditions $Ex(0-)$, of the GSSS (11) of the form

$$\mathcal{X}_T\hat{\xi}(0-) = [\mathcal{X}_T\ \bar{\mathcal{X}}_T] \begin{bmatrix} C & \\ CJ & \\ \vdots & 0_{qr,\mu} \\ CJ^{q-1} & \\ & C_\infty J_\infty^{q-1} \\ 0_{qr,n} & \vdots \\ & C_\infty J_\infty \\ & C_\infty \end{bmatrix} Ex(0-),$$

$$Ex(0-) = \begin{bmatrix} x_1(0-) \\ J_\infty x_2(0-) \end{bmatrix} = \begin{bmatrix} J^{q-1}B, J^{q-2}B, \ldots, B & 0_{n,q\mu} \\ 0_{\mu,qn} & B_\infty, J_\infty B_\infty, \ldots, J_\infty^{q-1}B_\infty \end{bmatrix} \begin{bmatrix} \mathcal{X}_T \\ \bar{\mathcal{X}}_T \end{bmatrix} \hat{\xi}(0-) \tag{14}$$

where

$$\bar{\mathcal{X}}_T := \begin{bmatrix} \mathcal{T}_0 & \mathcal{T}_1 & \cdots & \mathcal{T}_{q-1} \\ 0 & \mathcal{T}_0 & \cdots & \mathcal{T}_{q-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathcal{T}_0 \end{bmatrix}, \ \mathcal{X}_T := \begin{bmatrix} \mathcal{T}_q & 0 & \cdots & 0 \\ \mathcal{T}_{q-1} & \mathcal{T}_q & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ \mathcal{T}_1 & \mathcal{T}_2 & \cdots & \mathcal{T}_q \end{bmatrix}, \ \hat{\xi}(0-) := \begin{bmatrix} \xi(0-) \\ \xi^{(1)}(0-) \\ \vdots \\ \xi^{(q-1)}(0-) \end{bmatrix}. \tag{15}$$ □

Theorems 1 and 2 now allow the transfer of the solution and all the state reachability, controllability and observability spaces of the easily studied equivalent GSSS (11) to the solution and pseudostate reachability, controllability and observability spaces of the PMD (10).

Thus using the GSSS (2) and substituting the equivalent GSSS (11), i.e., $E_1 = J, B_1 = B\mathcal{U}, E_2 = J_\infty, B_2 = B_\infty \mathcal{U}, C_1 = \mathcal{V}C, C_2 = \mathcal{V}C_\infty$ we may apply the bijective map (13a) and obtain the solution of the PMD (10) as follows.

Theorem 3. The complete solution $\xi_{\text{com}}(t)$, of the PMD (10) is given as

$$\xi_{\text{com}}(t) = [C\ C_\infty] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \xi_1(t) + \xi_2(t) \tag{16a}$$

where

$$\xi_1(t) = Ce^{Jt}x_1(0-) + \int_{0-}^{t} Ce^{J(t-\tau)}B\mathcal{U}u(\tau)\, \mathrm{d}\tau$$

$$\xi_2(t) = \sum_{i=0}^{\eta-1} C_\infty J_\infty^i B_\infty \mathcal{U} u^{[i]}(t) + \sum_{i=0}^{\eta-2} \delta^{(i)}(t) C_\infty J_\infty^i \Big[(-1)J_\infty x_2(0-) + \sum_{j=0}^{\eta-2-i} J_\infty^{j+1} B_\infty \mathcal{U} u^{[j]}(0)\Big] \tag{16b}$$

where $u^{[i]}(t)$ denotes the ith regular derivative of $u(t)$, η is the degree of nilpotency of J_∞ and $x_1(0-), J_\infty x_2(0-)$ are defined in terms of $\hat{\xi}(0-)$ in Theorem 2. □

Definition 3 The space of initial values

$$\mathcal{A}_i := \left\{ \hat{\xi}(0-) : \Big[(-1)J_\infty x_2(0-) + \sum_{j=0}^{\eta-2-i} J_\infty^{j+1} B_\infty \mathcal{U} u^{[j]}(0)\Big] = 0,\ i = 0, \ldots \eta - 2 \right\} \tag{17}$$

for a given $u(t)$, are called *the set of impulse free initial values* for the PMD (10) where $x_1(0-)$, $J_\infty x_2(0-)$ are defined in terms of $\hat{\xi}(0-)$ in Theorem 2.. □

With this background we can now define pseudostate reachability and controllability and observability for the PMD (10).

Definition 4 Given a point $\xi_0 = \xi(0-) \in \mathcal{A}_i$, a point $\xi_T \in \mathbb{R}^r$ is *pseudostate reachable* (PR) from ξ_0 if there exists a $u(t)$ and $T > 0$ such that $\xi_{\text{com}}(t)$ is continuously differentiable on [0,T] and $\xi_{\text{com}}(T) = \xi_T$. If $\xi_0 = \xi(0-) \neq 0$ and $\xi_{\text{com}}(T) = \xi_T = 0 \in \mathbb{R}^r$, i.e., if the "origin" $0 \in \mathbb{R}^r$ is PR from ξ_0 then we say that ξ_0 is *pseudostate controllable* (PC). A point $\xi_T \in \mathbb{R}^r$ is *pseudostate observable* (PO) if the evolution of $\xi(T)$ for $T \geq 0$ may be determined using only knowledge of $u(T)$ and $y(T)$ for $T \geq 0$. □

Let now $\mathbf{R}_\mathrm{p}$ be the complete set of $\xi_T \in \mathbb{R}^r$ PR from $\xi_0 = \xi(0-) \in \mathcal{A}_i$ and let $\tilde{\mathbf{R}}_\mathrm{p}$ be the complete set of $\xi_T \in \mathbb{R}^r$ PC from $\xi_0 = \xi(0-) \in \mathcal{A}_i$.

Theorem 4.

(a) The *PR subspace* of (10) is given by $\mathbf{R}_\mathrm{p} = [C, C_\infty]\{< J|B\mathcal{U} > \oplus < J_\infty|B_\infty\mathcal{U} >\}$ and thus every $\xi_T \in \mathbb{R}^r$ is SR iff

$$\operatorname{rank}_{\mathbb{R}}[C, C_\infty] \begin{bmatrix} Q_{1p} & 0_{n,(\hat{q}_r+1)\mu} \\ 0_{\mu,nm} & Q_{2p} \end{bmatrix} = r, \tag{18}$$

(b) The *PC subspace* is given by $\bar{\mathrm{R}}_{\mathrm{p}} = [C, C_\infty]\{< J|B\mathcal{U} > \oplus [\ < J_\infty|B_\infty\mathcal{U} > + \ker J_\infty\ ]\ \}$ and thus every $\xi_0 \in \mathbb{R}^r$ is PC iff

$$\operatorname{rank}[C, C_\infty]\left[\begin{array}{c|c} Q_{1p} & 0_{n,(\hat{q}_r+1)+\sigma} \\ \hline 0_{\mu,nm} & Q_{2p}, M(J_\infty) \end{array}\right] = r, \tag{19}$$

(c) Every $\xi_T \in \mathbb{R}^r$ is PO iff

$$\operatorname{rank}_{\mathbb{R}}\left[\begin{array}{c} \mathcal{V}C \\ \mathcal{V}CJ \\ \vdots \\ \mathcal{V}CJ^{n-1} \end{array}\right] = n, \tag{20}$$

where $Q_{1p} := [B\mathcal{U}, JB\mathcal{U}, \ldots, J^{n-1}B\mathcal{U}] \in \mathbb{R}^{n\times nm}$, $Q_{2p} := [B_\infty\mathcal{U}, J_\infty B_\infty\mathcal{U}, \ldots, J_\infty^{\hat{q}_r}B_\infty\mathcal{U}] \in \mathbb{R}^{\mu\times(\hat{q}_r+1)m}$ are the finite and infinite pseudostate reachability matrices respectively of the PMD (10) and $M(J_\infty) \in \mathbb{R}^{\mu\times\varsigma}$, $\varsigma = \mu - \varepsilon$, $\varepsilon = \operatorname{rank}_{\mathbb{R}} J_\infty$, a basis matrix for $\ker J_\infty$. □

Thus it is evident from the above that pseudostate reachability of the PMD (10) implies its pseudostate controllablity but the converse is not true.

It has therefore been shown that a recently developed algorithm ([1]) for obtaining an "equivalent" generalised state-space representation of a linear multivariale system whose dynamics are expressed by PMDs may be used to present the solution of a PMD and propose certain pseudostate reachability, controllability and observability criteria. Thus satisfying extensions to the results of the solution in terms of the regular derivative of $u(t)$, state reachability, controllability and observability ([8], [3], [9], [2]) to the general PMD case have been developed. Proofs have been omitted because of lack of space but may be found in [10].

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