International Journal of Control
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tcon20

The structure of multivariable systems under the action of constant output feedback control laws

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Version of record first published: 08 Mar 2007.

INT. J. CONTROL, 1978, VOL. 27, NO. 2, 261–269

The structure of multivariable systems under the action of constant output feedback control laws

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The structure of linear, time-invariant, completely controllable and observable multivariable systems under the action of constant output feedback control laws is studied. A structure theorem is stated and proved. Following Dickinson (1976), the notion of covariant output feedback control laws is introduced. It is shown that these results can be used to investigate the problems concerning (i) our ability to alter some of the Popov (1972) invariants by employing constant output feedback control, and (ii) the asymptotic behaviour of the closed loop poles under the variation of the output feedback gain matrix.

1. Introduction

Firstly an expression for the zeros of a transfer function matrix $G(s)$ in terms of the parameters $(\tilde{A}, \tilde{B}, \tilde{C})$ of a corresponding minimal state-space model in the Brunovski (1966), Luenberger (1967) controllable canonical form is established. Then a structure theorem for multivariable systems under constant output feedback is given. In the light of these results and the notion of covariant output feedback control laws, two problems are studied. These concern (i) the possibility of changing some of the Popov (1972) invariants by employing constant output feedback, and (ii) the asymptotic behaviour of the closed-loop poles under the variation of the constant output feedback. The answers to both questions lead to generalizations in the multivariable case of some classical root locus method ideas.

2. The zeros of G(s)

Consider a linear, time-invariant, completely controllable and observable multivariable system in state-space form

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where $A \in R^{n \times n}$, $B \in R^{n \times m}$, $C \in R^{r \times n}$. Let $G(s) = C(sI - A)^{-1}B$ be the $r \times m$ transfer function matrix corresponding to the above system and let

$$\begin{aligned} M(s) &= \{\text{diag}\,(\epsilon_i(s)/\psi_i(s))0_{r,m-r}\} \quad & m > r) \\ &= \text{diag}\,(\epsilon_i(s)/\psi_i(s)) & (m = r) \\ &= \begin{bmatrix} \text{diag}\,(\epsilon_i(s)/\psi_i(s)) \\ \\ 0_{r-m,m} \end{bmatrix} & (m < r) \end{aligned}$$

denote the MacMillan form of $G(s)$ (Rosenbrock 1970).

Received 29 October 1976.

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The zeros of the transfer function matrix $G(s)$ are defined by Rosenbrock (1970, 1973) as the zeros of the numerator polynomials $\epsilon_i(s)$, $i=1, 2, \ldots, \min(m, r)$ in the MacMillan form $M(s)$ of $G(s)$.

Let

$$G(s)=N(s)D^{-1}(s) \tag{3}$$

be a (non-unique) ' matrix fraction ' factorization of $G(s)$ (Rosenbrock 1970) with $N(s)$ and $D(s)$ relatively right prime polynomial matrices of appropriate dimensions, and $D(s)$ non-singular over the rational field.

Wolovich (1973) showed that all ' numerator ' $r \times m$ polynomial matrices $N(s)$ satisfying eqn. (3) have the same Smith form : $\epsilon(s)$, which is given by

$$\left.\begin{aligned} \epsilon(s) &= (\operatorname{diag}(\epsilon_i(s))0_{r,m-r}) && (m>r) \\ &= \operatorname{diag}(\epsilon_i(s)) && (m=r) \\ &= \begin{bmatrix} \operatorname{diag}(\epsilon_i(s)) \\ 0_{r-m,m} \end{bmatrix} && (m<r) \end{aligned}\right\} \tag{4}$$

So in the light of the above result, Wolovich (1973) gave an equivalent definition for the zeros of $G(s)$.

Definition 1 (Wolovich 1973)

The zeros of $G(s)$ are the zeros of the invariant polynomials $\epsilon_i(s)$, $i=1, 2, \ldots, \min(m, r)$, of the Smith form $\epsilon(s)$ of any ' numerator ' polynomial matrix $N(s)$ of $G(s)$. Assume now that $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, rank $\tilde{B}=m$ is a minimal realization of $G(s)$ in the controllable canonical form of Brunovski (1966) and Luenberger (1967).

Then if $k_1 \geqslant k_2 \geqslant \ldots \geqslant k_m \geqslant 1$ are the controllability indices of $\{\tilde{A}, \tilde{B}\}$ and $d_i \triangleq \sum_{j=1}^{i} k_j$, $i=1, 2, \ldots, m$ by virtue of the structure theorem of Wolovich and Falb (1969), and Wolovich (1974), $G(s)$ can be written as :

$$G(s)=\tilde{C}S(s)\{\tilde{B}_m^{-1}\delta_0(s)\}^{-1} \tag{5}$$

where $S(s)=\text{block diag }\{\hat{s}_1, \ldots, \hat{s}_m\}$, $\hat{s}_i^{\tau}=\{1s \; \ldots \; s^{k_i-1}\}$; $\delta_0(s)=\operatorname{diag}\{s^{k_1}, \ldots, s^{k_m}\}-\tilde{A}_mS(s)$; $\tilde{A}_m$ is an $m \times n$ matrix consisting of the m ordered d_i rows of A ; and $\tilde{B}_m$ is an $m \times m$ matrix consisting of the m ordered d_i rows of $\tilde{B}$. It is evident from eqn. (5) that the matrix product $\tilde{C}S(s)$ is an $r \times m$ polynomial matrix which can be considered as a ' numerator ' $N(s)$ of $G(s)$ (Wolovich 1974). From the above analysis and Wolovich's (1973) definition of the zeros of $G(s)$ we can state the following :

Proposition 1

Let $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ be the minimal realization of $G(s)$ in the Brunovski–Luenberger controllable canonical form. The zeros of $G(s)$ solely depend on $\tilde{C}$ and on the controllability indices k_i, $i=1, 2, \ldots, m$ for the pair $\{\tilde{A}, \tilde{B}\}$. They are given by the zeros of the invariant polynomials $\epsilon_i(s)$, $i=1, 2, \ldots, \min(m, r)$ of the Smith form of the matrix product $\tilde{C}S(s)=N(s)$.

Assume now that $r=m$, then according to definition 1, the number q of the zeros of $G(s)$ must be equal to degree $\prod_{i=1}^{m} \epsilon_i(s)$ and from theorem 4.1 of Rosenbrock (1970),

$$q \leqslant n-m.$$

An algorithm to calculate the zeros of $G(s)$ can now be summarized as follows :

(1) Calculate the Brunovski–Luenberger controllable canonical realization of $G(s)$. In this process the controllability indices k_i, $i=1, 2, \ldots, m$ are also calculated.

(2) Having obtained the controllability indices k_i, $i=1, 2, \ldots, m$ form $S(s)$ and calculate $N(s)=\tilde{C}S(s)$.

(3) If $r=m$ then calculate det $N(s)=z(s)$ and determine the zeros of $G(s)$ as the roots of $z(s)=0$, otherwise go to step (4).

(4) If $r \neq m$, determine the Smith form $\epsilon(s)$ of $N(s)$. Then the zeros of $G(s)$ are equal to the zeros of $\epsilon_i(s)$, $i=1, 2, \ldots, \min(m, r)$.

3. A structure theorem

We now give a structure theorem for linear, time-invariant, completely controllable and observable systems under the action of constant output feedback control laws. This theorem, which is our main result, is proved using the structure theorem of Wolovich and Falb (1969) and Wolovich (1974) and can be seen as an analogue to that theorem when constant output (rather than state) feedback is considered.

Structure theorem

Let $(\tilde{A}, \tilde{B}, \tilde{C})$ with rank $\tilde{B}=m$ be the minimal realization of $G(s)$ in the Brunovski–Luenberger controllable canonical form. Let the output feedback be defined by $u=v-K_y y$, $K_y \in R^{m\times r}$. Then the transfer function matrix of the closed-loop system $G_k(s)$ can be written as :

$$G_k(s)=\tilde{C}S(s)(\tilde{B}_m{}^{-1}\delta_k(s))^{-1} \tag{6}$$

where

$$\delta_k(s)=\delta_0(s)+KN(s) \tag{7}$$

$$K=\tilde{B}_m K_y \tag{8}$$

and $N(s)=\tilde{C}S(s)$.

Proof

Let $K \in R^{m\times r}$ be arbitrary and write it in terms of its m rows $K_{di}^T \in R^{1\times r}$, $i=1, 2, \ldots, m$ as

$$K=\begin{bmatrix} K_{d_1}^T \\ \vdots \\ K_{d_m}^T \end{bmatrix} \tag{9}$$

With K_y given by eqn. (8), and using the fact that $\tilde{B}=\hat{B}\tilde{B}_m$ where $\hat{B}=$ block diag $\{\hat{b}_1, \ldots, \hat{b}_m\}$, $\hat{b}_i^{\tau}=\{00 \ldots 01\} \in R^{1 \times k_i}$, $i=1, 2, \ldots, m$, the closed-loop plant matrix $\bar{A}$ is (Vardulakis 1975, 1976)

$$\bar{A}=\tilde{A}-\tilde{B}K_y\tilde{C}=\tilde{A}-\hat{B}\tilde{B}_m\tilde{B}_m^{-1}K\tilde{C}=\tilde{A}-\hat{B}K\tilde{C}=\tilde{A}-K_z\tilde{C} \tag{10}$$

where $K_z=\hat{B}K$ and has the form

$$K_z=\begin{bmatrix} 0_{k_1-1,r} \\ K_{d_1}^{\tau} \\ \vdots \\ 0_{k_m-1,r} \\ K_{d_m}^{\tau} \end{bmatrix} \in R^{n \times r} \tag{11}$$

So from eqns. (10) and (11), $\bar{A}$ has the same block-structure as $\tilde{A}$. Now let $\bar{A}_m$ be the $m \times n$ matrix consisting of the m ordered d_i rows of $\bar{A}$, then eqn. (10) gives

$$\bar{A}_m=\tilde{A}_m-K\tilde{C} \tag{12}$$

and our structure theorem follows directly from the structure theorem of Wolovich and Falb (1969) and Wolovich (1974), according to which $G_k(s)$ can be written as in eqn. (6) where

$$\begin{aligned} \delta_k(s) &= \operatorname{diag}\{s^{k_1}, \ldots, s^{k_m}\}-\bar{A}_m S(s) \\ &= \operatorname{diag}\{s^{k_1}, \ldots, s^{k_m}\}-\tilde{A}_m S(s)+K\tilde{C}S(s) \\ &= \delta_0(s)+KN(s) \end{aligned}$$

It is also evident from eqn. (6) (see also Wolovich and Falb 1969 and Wolovich 1974) that if $\Delta_k(s)=\det(sI-\bar{A})$ is the closed-loop characteristic polynomial then

$$\Delta_k(s)=\det \delta_k(s) \tag{13}$$

Furthermore, defining $D_0(s)=\tilde{B}_m^{-1}\delta_0(s)$ and $D_k(s)=\tilde{B}_m^{-1}\delta_k(s)$ it follows from eqns. (7) and (8) that

$$D_k(s)=D_0(s)+K_yN(s)$$

So the expression for $G_k(s)$ can be written more succinctly as

$$G_k(s)=N(s)(D_0(s)+K_yN(s))^{-1}$$

4. Covariant output feedback control laws

Let A, B, C be a controllable and observable triplet and let $\tilde{A}=QAQ^{-1}$, $\tilde{B}=QB$, $\tilde{C}=CQ^{-1}$ be its Brunovski–Luenberger controllable canonical form. As described by Denham (1974) and Dickinson (1976),

$$\tilde{A}=\begin{bmatrix} A_{11} & \cdots & A_{1m} \\ \vdots & & \vdots \\ A_{m1} & \cdots & A_{mm} \end{bmatrix} \quad \tilde{B}=\begin{bmatrix} B_1 \\ \vdots \\ B_m \end{bmatrix}$$

and the blocks A_{ii} and A_{ij} have the form

$$A_{ii}=\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \\ x & x & x & \dots & x \end{bmatrix} \quad A_{ij}=\begin{bmatrix} & & & & 0 & & & \\ x & x & \dots & x & 0 & 0 & \dots & 0 \end{bmatrix} \quad (i \neq j)$$

where ' x ' denotes a possibly non-zero entry. That is, for $i \neq j$, the last row of A_{ij} can have possibly non-zero entries only in the first min (k_i, k_j) columns, and the remaining (if any) columns are occupied by zeros. Following Dickinson (1976) we call these zeros, when they exist, ' sacred zeros '. The block B_i has zero entries except that in row k_i it has a ' 1 ' in column i and an ' x ' in every column $j > i$ for which $k_j < k_i$. The matrix $\tilde{C}$ has no special form and it contains only possibly non-zero entries ' x '.

Popov (1972) showed that the possibly non-zero entries ' x ' of $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and the controllability indices k_i, $i=1, 2, \dots, m$, constitute a complete system of independent invariants for the triplet A, B, C under similarity transformations.

We now introduce the notion of covariant output feedback control laws. This notion was motivated by the results on covariant state feedback control laws, examined by Dickinson (1976).

Definition 2

An output feedback law K_y is called covariant (relative to A, B, C) if $Q(A-BK_yC)Q^{-1}$, QB, CQ^{-1} is in the Brunovski–Luenberger controllable canonical form.

From

$$Q(A-BK_yC)Q^{-1}=\tilde{A}-K_z\tilde{C}=\bar{A}$$

(see eqn. (10)) we have—

Proposition 2

K_y (as defined by eqn. (8)), is covariant (relative to A, B, C) if and only if K is such that $\bar{A}_m=\tilde{A}_m-K\tilde{C}$ has zeros in the positions corresponding to the positions of the ' sacred zeros ' (if any) of the blocks A_{ij} $(i \neq j)$ of $\tilde{A}$.

When all the controllability indices k_i, $i=1, 2, \dots, m$ are equal $(k_i=n/m)$ then there are no ' sacred zeros ' and so we have—

Proposition 3

If A, B are such that the controllability indices k_i, $i=1, 2, \dots, m$ are all equal, then every K_y is covariant (relative to A, B, C).

Considering the results of Popov (1972) mentioned earlier, and eqn. (12), the control theoretical significance of covariant output feedback control laws becomes clear. If K_y is covariant (relative to A, B, C), then eqn. (12) tells us exactly what control we have in altering the Popov (1972) invariants ' x ' associated with $\tilde{A}$, by employing constant output feedback. Furthermore, it gives us the invariants associated with the closed-loop plant matrix $\bar{A}$.

Remark

We notice that our structure theorem is valid whether or not K_y is covariant (relative to A, B, C). Wolovich's (1974) structure theorem under state feedback is also valid for covariant or non-covariant state feedback laws Dickinson, (1976). This observation was not made by Wolovich (1974), who proved the above theorem assuming that all state feedback control laws are covariant (relative to A, B), (Dickinson 1976).

If K_y is not covariant (relative to A, B, C), then the closed-loop system $\tilde{A}-K_z\tilde{C}$, $\tilde{B}$, $\tilde{C}$, is not in the Brunovski–Luenberger controllable canonical form. We call such a form a controllable pseudo-canonical (c.p.) form. If $\tilde{A}-K_z\tilde{C}$, $\tilde{B}$, $\tilde{C}$ is in the c.p. form, then there exists a non-singular matrix $L \in R^{n\times n}$ such that $L(\tilde{A}-K_z\tilde{C})L^{-1}$, $L\tilde{B}$, $\tilde{C}L^{-1}$ is in the Brunovski–Luenberger controllable canonical form. In such a case $L\tilde{B}=\tilde{B}$. Thus every output feedback law (covariant or not) preserves the canonical form of the input matrix.

5. Asymptotic behaviour of the closed-loop poles

It has been established by Rosenbrock (1970, 1973), that the zeros of a transfer function matrix are invariant under constant output feedback. So in the following, by ' zeros of $G(s)$ ' we also mean the zeros of $G_k(s)$.

We will now state and prove a result concerning the asymptotic behaviour of the closed-loop poles under the variation of the output feedback matrix K_y. This result was stated recently by Kouvaritakis and MacFarlane (1976), who originally examined the asymptotic behaviour of the closed-loop poles under the restricting assumption that the feedback gain matrix K_y has the specific form $K_y=kI$, and $k\to\infty$.

Proposition 4

Let $r=m$. As the gain matrix K (and also, according to eqn. (8), the output feedback matrix K_y) changes from a zero matrix to a matrix whose elements (K_{ij}) change in such a way that $\|K\|$ tends to infinity, the closed-loop poles move in the s-plane as follows :

They start, for $K=0$, from the n open-loop poles, and as (K_{ij}) change so that $\|K\|\to\infty$, $n-q$ poles become arbitrarily large in magnitude while the remaining q poles tend to the q zeros of $G(s)$.

Proof

Notice that the open-loop characteristic polynomial is $\Delta_0(s)=\det\,\delta_0(s)$ (Wolovich 1974), and also that if $r=m$ according to proposition 1 the zeros of $G(s)$ are equal to the q zeros of $\det N(s)=0$. So proposition 4 follows directly from eqns. (7) and (13).

6. Example

Let us consider the controllable and observable system (Kouvaritakis and MacFarlane 1976) with :

$$A=\begin{bmatrix}-1 & 1 & 3 & -2\\ 0 & -1 & -1 & -1\\ 0 & 1 & -3 & -1\\ 0 & 3 & -1 & -5\end{bmatrix}\quad B=\begin{bmatrix}1 & 0\\ 1 & 2\\ 1 & 1\\ 2 & 2\end{bmatrix}\quad C=\begin{bmatrix}1 & -1 & 3 & 0\\ 0 & -1 & -3 & 2\end{bmatrix}$$

Determining the transformation matrix Q (Brunovski 1966, Luenberger 1967), and its inverse Q^{-1} (Jordan and Sridhar 1973)

$$Q=\begin{bmatrix} 0\cdot333333 & 0\cdot333333 & 0\cdot666666 & -0\cdot666666 \\ -0\cdot333333 & -1\cdot333333 & -0\cdot66666 & 1\cdot66666 \\ 0 & 0 & -2 & 1 \\ 0 & 1 & 5 & -3 \end{bmatrix}$$

$$Q^{-1}=\begin{bmatrix} 4 & 1 & 1 & 0 \\ 1 & 1 & 5 & 2 \\ 1 & 1 & 2 & 1 \\ 2 & 2 & 5 & 2 \end{bmatrix}$$

we obtain the controllable canonical form for the above system as :

$$\tilde{A}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -4 & -5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -6 & -5 \end{bmatrix} \quad \tilde{B}=\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \tilde{C}=\begin{bmatrix} 6 & 3 & 2 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

Hence $k_1=2$, $k_2=2$ and

$$N(s)=\tilde{C}S(s)=\begin{bmatrix} 6 & 3 & 2 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 0 \\ 0 & 1 \\ 0 & s \end{bmatrix}=\begin{bmatrix} 6+3s & 2+s \\ 0 & -(1+s) \end{bmatrix}$$

so that $\det N(s)=-(6+3s)(1+s)=0$ gives the zeros of $\hat{G}(s)$ as $s=-2$, $s=-1$.

Furthermore :

$$\delta_0(s)=\operatorname{diag}\{s^{k_1}, s^{k_2}\}-\tilde{A}_m S(s)=\begin{bmatrix} s^2 & 0 \\ 0 & s^2 \end{bmatrix}-\begin{bmatrix} -4 & -5 & 0 & 0 \\ 0 & 0 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 0 \\ 0 & 1 \\ 0 & s \end{bmatrix}$$

$$=\begin{bmatrix} s^2+5s+4 & 0 \\ 0 & s^2+5s+6 \end{bmatrix}$$

and for $K=(k_{ij})$,

$$KN(s)=\begin{bmatrix} k_{11}(6+3s) & k_{11}(s+2)-k_{12}(s+1) \\ k_{21}(6+3s) & k_{21}(s+2)-k_{22}(s+1) \end{bmatrix}$$

so that eqn. (7) gives : $\delta_k(s)=(\delta_{k_{ij}}(s))$ with

$$\begin{aligned} \delta_{k_{11}}(s) &= s^2+5s+4+k_{11}(6+3s) \\ \delta_{k_{12}}(s) &= k_{11}(s+2)-k_{12}(s+1) \\ \delta_{k_{21}}(s) &= k_{21}(6+3s) \\ \delta_{k_{22}}(s) &= s^2+5s+6+k_{21}(s+2)-k_{22}(s+1) \end{aligned}$$

For $k_{2i}=0$, $\delta_k(s)$ becomes upper triangular, so that

$$\Delta_k(s)=\det\,\delta_k(s)=\delta_{k_{11}}(s)\,.\,\delta_{k_{22}}(s)=0 \tag{14}$$

From eqn. (14) we obtain

$$\delta_{k_{11}}(s)=s^2+5s+4+k_{11}(6+3s)=0 \tag{15}$$

$$\delta_{k_{22}}(s)=s^2+5s+6-k_{22}(s+1)=0 \tag{16}$$

Equations (15) and (16) can be written in the ' root-locus form ' :

$$\begin{aligned} -1 &= R_1(s+2)/(s+1)(s+4) \\ -1 &= R_2(s+1)/(s+2)(s+3) \end{aligned}$$

where $R_1=3k_{11}$, $R_2=-k_{22}$ give rise to the root loci in Figs 1 and 2.

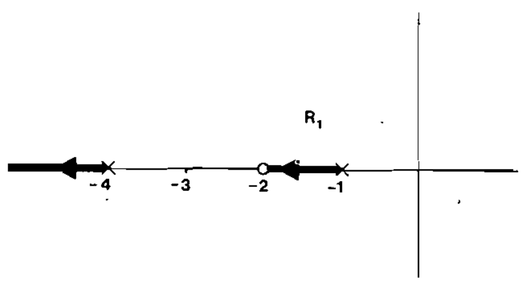

Figure 1.

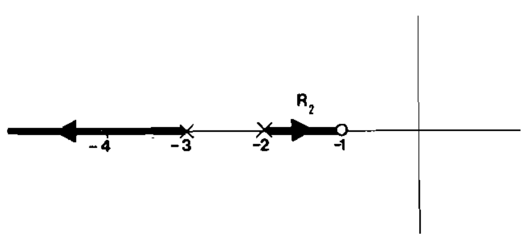

Figure 2.

7. Conclusions

The structure theorem developed in this paper shows more clearly than a previous attempt (Vardulakis 1976), that the classical (scalar) 'root-locus condition'

$$1+\frac{K\text{ (numerator of } G(s))}{\text{denominator of } G(s)}=0$$

is a special case (when $r=m=1$) of the more general matrix equation (7). Putting the matrices which define the state-space model in the controllable canonical form and considering the problem of selecting an output feedback constant gain natrix K_y such that, after feedback, the resulting closed-loop system has a desired plant matrix $\bar{A}$ (Vardulakis 1976, Sinswat and Fallside 1976), we can make the following remarks :

(1) If the constant output feedback is given by eqn. (8), then the family of all attainable closed-loop plant matrices $\bar{A}$ is given by eqn. (10).

(2) The above problem of obtainability of a certain plant matrix is, in general, different from the pole assignment problem using constant output feedback. Both these problems coincide only in the single-input, single-output case, where the root locus uniquely defines both the attainable poles and the closed-loop plant matrices in the companion (canonical) form.

ACKNOWLEDGMENTS

The author would like to thank the U.K. Science Research Council for financial support. Acknowledgment is also made of many useful discussions with M. J. Denham, and to Professor D. Q. Mayne for his helpful comments.

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