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Necessary and sufficient condition for stability robustness under known additive perturbation

A. I. G. VARDULAKIS†

A necessary and sufficient condition is derived for the stability robustness of a unity feedback closed-loop system involving a strictly proper plant P_0 and a stabilizing compensator C_0 under the assumption that the plant P_0 is perturbed to $P_0 + \Delta P_0$ where ΔP_0 is a known strictly proper rational matrix.

Notation

$\mathbb{R}$	the field of reals
$\mathbb{C}$	the field of complex numbers
$\mathbb{C}_0^- :=$	$\{s \in \mathbb{C}, \operatorname{Re}(s) < 0\}$
$\mathbb{C}^+ :=$	$\{s \in \mathbb{C}, \operatorname{Re}(s) \geqq 0\}$
$\bar{\mathbb{C}}^+ :=$	$\mathbb{C}^+ \cup \{\infty\}$
$\mathbb{R}(s)$	the field of rational functions with coefficients in $\mathbb{R}$
$\mathbb{R}_{pr}(s)$	the ring of proper rational functions
Ω	any subset of $\mathbb{C}$ which is symmetrically located with respect to $\mathbb{R}$ and which excludes at least one point $-\alpha \in \mathbb{R}$ $(\alpha > 0)$
$\bar{\Omega} :=$	$\Omega \cup \{\infty\}$
S	the euclidean ring of rational functions $t(s) \in \mathbb{R}(s)$ which have no poles in $\bar{\Omega}$, i.e. of 'proper and Ω-stable' rational functions.
$k^{p \times m}$	the set of $p \times m$ matrices with elements in k if k is a set.

1. Introduction

Let $\Sigma(P_0)$ be a linear multivariable system which is free of unstable hidden modes and whose input–output behaviour is described by a $p \times m$ strictly proper rational transfer function matrix P_0 (the 'plant'). Consider now the closed-loop unity feedback system $\Sigma_{cl}(P_0, C_0)$ of Fig. 1 which involves a 'stabilizing compensator' C_0 such that $\Sigma_{cl}(P_0, C_0)$ is internally stable and the closed-loop transfer function matrix

$$H_{yu}(P_0, C_0): \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \to \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{1}$$

has arbitrary desired poles in $\mathbb{C}_0^-$. Now let the nominal plant P_0 be perturbed to $P_0 + \Delta P_0 =: P$ where ΔP_0 is a known $p \times m$ proper rational matrix. In this paper we give a simple necessary and sufficient condition that has to be satisfied by ΔP_0 so that if the additively perturbed system $\Sigma(P_0 + \Delta P_0)$ is also free of unstable hidden modes, the closed-loop system $\Sigma_{cl}(P_0 + \Delta P_0, C_0)$ of Fig. 2 is also internally stable.

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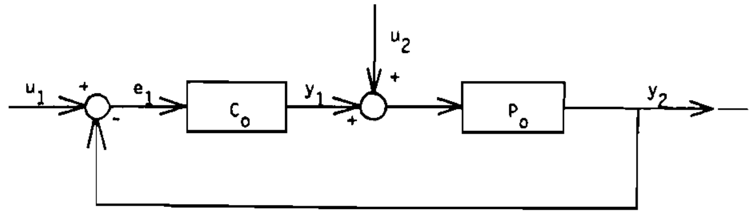

Figure 1.

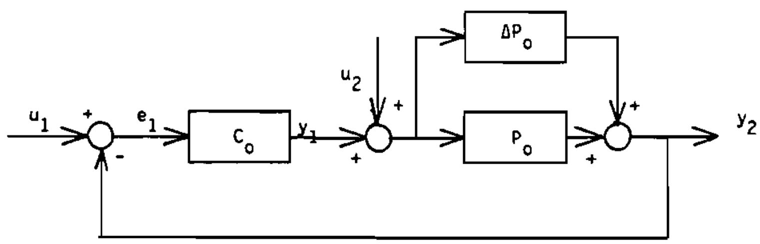

Figure 2.

2. Set of plants stabilizable by a compensator which stabilizes a nominal plant P_0

Let $\Sigma(P_0)$ be a linear, time-invariant multivariable system which is free of unstable hidden modes and whose (nominal) transfer function matrix is $P_0 \in \mathbb{R}_{pr}^{p\times m}(s)$ and let $P_0 = A_1^{-1}B_1 = B_2A_2^{-1}$ be left and right coprime in $\bar{\Omega}$ 'fractional representations' of P_0, i.e. let the matrices $[A_1 \quad B_1] \in S^{p\times(p+m)}$ and $\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} \in S^{(m+p)\times m}$ be respectively S-right and left invertible. It is then known (Desoer *et al.* 1980, Vidyasagar 1985, Vidyasagar *et al.* 1982, Saeks and Murray 1981, Youla *et al.* 1976, Callier and Desoer 1982) that another system $\Sigma'(C_0)$ (which is also free of unstable hidden modes) with transfer function matrix $C_0 \in \mathbb{R}_{pr}^{m\times p}(s)$ is a 'stabilizing compensator' for $\Sigma(P_0)$ such that the unity feedback closed-loop system denoted by $\Sigma_{cl}(P_0, C_0)$ of Fig. 1 is internally stable (i.e. all it modes, including the uncontrollable and unobservable ones lie in $\mathbb{C}_0^-$) iff $|I_p + P_0(\infty)C_0(\infty)| \neq 0$ and the transfer function matrix $H_{yu}(P_0, C_0): [u_1 \quad u_2]^T \rightarrow [y_1 \quad y_2]^T$ is an element of $S^{(m+p)\times(m+p)}$ where $\bar{\Omega} \equiv \bar{\mathbb{C}}^+$. It is also well known that if C_0 is a stabilizing compensator for $\Sigma(P_0)$ then C_0 has a unique left and right coprime in $\bar{\Omega}$ fractional representations

$$C_0 = D_1^{-1}N_1 = N_2D_2^{-1} \tag{2}$$

satisfying the Bezout identity

$$\begin{bmatrix} D_1 & N_1 \\ -B_1 & A_1 \end{bmatrix}\begin{bmatrix} A_2 & -N_2 \\ B_2 & D_2 \end{bmatrix} = \begin{bmatrix} I_m & 0 \\ 0 & I_p \end{bmatrix} \tag{3}$$

Multiplying (3) on the left and right by the S-unimodular matrices $\begin{bmatrix} I_m & W \\ 0 & I_p \end{bmatrix}$ and

$\begin{bmatrix} I_m & -W \\ 0 & I_p \end{bmatrix}$, respectively, where $W \in S^{m \times p}$ such that $|[D_1 - WB_1](\infty)| \neq 0$ and $|[D_2 - B_2 W](\infty)| \neq 0$, we obtain the identity

$$\begin{bmatrix} D_1 - WB_1 & N_1 + WA_1 \\ -B_1 & A_1 \end{bmatrix} \begin{bmatrix} A_2 & -(N_2 + A_2 W) \\ B_2 & D_2 - B_2 W \end{bmatrix} = \begin{bmatrix} I_m & 0 \\ 0 & I_p \end{bmatrix} \tag{4}$$

which, due to the above results, shows clearly that the set $\phi(P_0)$ of all stabilizing compensators C_W of $\Sigma(P_0)$ is parametrized by C_0 and W and is given by

$$\begin{aligned} \phi(P_0) = \{C_W = (D_1 - WB_1)^{-1}(N_1 + WA_1) = (N_2 + A_2 W)(D_2 - B_2 W)^{-1} \mid W \in S^{m \times p} \\ \text{such that} \quad |[D_1 - WB_1](\infty)| \neq 0, |[D_2 - B_2 W](\infty)| \neq 0\} \end{aligned} \tag{5}$$

(See the references cited above.)

Consider now the nominal plant P_0 a (nominal) stabilizing compensator $C_0 \in \phi(P_0)$ and denote by $\psi(C_W)$ the set of all 'plants' stabilizable by a compensator $C_W \in \phi(P_0)$. By duality to the above characterization of $\phi(P_0)$ we can parametrize $\psi(C_W)$. Then we have the following result.

Proposition 1

Let $P_0 = A_1^{-1} B_1 = B_2 A_2^{-1} \in \mathbb{R}_{pr}^{p \times m}(s)$, $C_0 = D_1^{-1} N_1 = N_2 D_2^{-1} \in \mathbb{R}^{m \times p}(s)$, a nominal stabilizing compensator of $\Sigma(P_0)$, and $C_W \in \phi(P_0)$. Then the set $\psi(C_W)$ of all 'plants' stabilizable by C_W which stabilizes P_0 is given by:

$$\begin{aligned} \psi(C_W) = \psi(P_0, C_0) = \{P_{Q,W} &= [A_1 + Q(N_1 + WA_1)]^{-1}[B_1 - Q(D_1 - WB_1)] \\ &= [B_2 - (D_2 - B_2 W)Q][A_2 + (N_2 + A_2 W)Q]^{-1}, \\ & Q \in S^{p \times m}, \quad W \in S^{m \times p}\} \end{aligned} \tag{6}$$

$$|[A_1 + Q(N_1 + WA_1)](\infty)| \neq 0, \quad |[A_2 + (N_2 + A_2 W)Q](\infty)| \neq 0 \tag{7}$$

Proof

Multiplying (4) on the left and right respectively by the S-unimodular matrices $\begin{bmatrix} I_m & 0 \\ Q & I_p \end{bmatrix}$ and $\begin{bmatrix} I_m & 0 \\ -Q & I_p \end{bmatrix}$, where $Q \in S^{p \times m}$ and is such that conditions (7) are satisfied, we obtain the identity

$$\begin{bmatrix} D_1 - WB_1 & N_1 + WA_1 \\ -[B_1 - Q(D_1 - WB_1)] & A_1 + Q(N_1 + WA_1) \end{bmatrix} \begin{bmatrix} A_2 + (N_2 + A_2 W)Q & -(N_2 + A_2 W) \\ B_2 - (D_2 - B_2 W)Q & D_2 - B_2 W \end{bmatrix} = \begin{bmatrix} I_m & 0 \\ 0 & I_p \end{bmatrix} \tag{8}$$

which, owing to the above results clearly shows that the set $\psi(C_W)$ is given by (6).

□

Corollary 1

The set $\psi(C_0)$ of all plants stabilizable by $C_0 \in \phi(P_0)$ is given by

$$\psi(C_0) = \{P_Q = (A_1 + QN_1)^{-1}(B_1 - QD_1) = (B_2 - D_2Q)(A_2 + N_2Q)^{-1}, \quad Q \in S^{p\times m}\} \tag{6'}$$

$$|[A_1 + QN_1](\infty)| \neq 0, \quad |[A_2 + N_2Q](\infty)| \neq 0 \tag{7'}$$

Proof

Just put $W = 0$ in (6). □

We investigate now conditions under which the elements of the set $\psi(C_0)$ are proper rational matrices. To this end the next well-known lemma is in order.

Lemma 1 (Vardulakis and Karcanias 1983)

Let $T \in \mathbb{R}^{p\times m}(s)$ and let $D_L \in S^{p\times p}$, $N_L \in S^{p\times m}$ left coprime in $\bar{\Omega}$, $N_R \in S^{p\times m}$, $D_R \in S^{m\times m}$ right coprime in $\bar{\Omega}$ and such that $T = D_L^{-1}N_L = N_R D_R^{-1}$. Then

(i) $T \in \mathbb{R}_{pr}^{p\times m}(s)$ iff D_L, D_R are biproper or equivalently iff $|D_L(\infty)| \neq 0$, $|D_R(\infty)| \neq 0$.

Moreover if (i) holds true then T is strictly proper iff N_L, N_R are strictly proper or equivalently iff $N_L(\infty) = 0$, $N_R(\infty) = 0$.

If we assume now that P_0 is strictly proper, then from the above lemma: $|A_1(\infty)| \neq 0$, $|A_2(\infty)| \neq 0$, $B_1(\infty) = 0$, $B_2(\infty) = 0$ and hence from (3) we obtain $D_1(\infty)A_2(\infty) = I_m$, i.e. $|D_1(\infty)| = |A_2(\infty)|^{-1} \neq 0$ which again by virtue of Lemma 1 implies that $C_0 \in \mathbb{R}_{pr}^{m\times p}(s)$. Moreover

$$(D_1 - WB_1)(\infty) = D_1(\infty) - W(\infty)B_1(\infty) = D_1(\infty) \tag{9}$$

so that for every $W \in S^{m\times p}$, every $C_W \in \phi(P_0)$ will be proper. On the other hand, if P_0 is proper but not strictly proper then it is known that $W \in S^{m\times p}$ can be chosen so that C_W is proper or even strictly proper (Vidyasagar 1985). Of course a strictly proper compensator C_W also always exists in the case of a strictly proper plant P_0.

We look now more closely into the set $\psi(C_0)$. According to Lemma 1, if P_0 is proper then the matrices A_1 and A_2 are biproper and if either $Q \in S^{p\times m}$ and/or $C_0 = D_1^{-1}N_1 = N_2D_2^{-1}$ are chosen so that the product $QN_1 \in S^{p\times p}$ (or $N_2Q \in S^{m\times m}$) is not strictly proper, then the matrices $A_1 + QN_1$ and $A_2 + N_2Q$ appearing in (6′) might turn out to be non-biproper. Thus in general, and even if P_0 is strictly proper, the set $\psi(C_0)$ will also contain non-proper 'plants' P_0. On the other hand, if $Q \in S^{p\times m}$ is chosen to be strictly proper then $P_Q \in \mathbb{R}_{pr}^{p\times m}(s)$. If P_0 is strictly proper, then the strictly proper elements P_Q of $\psi(C_0)$ can be characterized by Q. We have the following result.

Proposition 2

Let $P_0 \in \mathbb{R}_{pr}^{p\times m}(s)$ be strictly proper with $P_0 = A_1^{-1}B_1 = B_2A_2^{-1}$ and let $C_0 \in \mathbb{R}_{pr}^{m\times p}(s)$ with $C_0 = D_1^{-1}N_1 = N_2D_2^{-1}$ such that the Bezout identity (3) is satisfied, i.e. let $C_0 \in \phi(P_0)$. Then

(i) for every $Q \in S^{p\times m}$ which is strictly proper

$$P_Q := (A_1 + QN_1)^{-1}(B_1 - QD_1) = (B_2 - D_2Q)(A_2 + N_2Q)^{-1} \in \psi(C_0) \tag{10}$$

and it is strictly proper.

(ii) Every strictly proper plant $P_Q \in \psi(C_0)$ can be expressed as in (10) for some strictly proper $Q \in S^{p\times m}$.

Proof

(i) From Corollary 1 it follows that for every strictly proper Q, $P_Q \in \psi(C_0)$. Moreover, we have

$$(A_1 + QN_1)(\infty) = A_1(\infty) + Q(\infty)N_1(\infty) = A_1(\infty) \tag{11}$$

$$(B_1 + QD_1)(\infty) = B_1(\infty) + Q(\infty)D_1(\infty) = 0 \tag{12}$$

By Lemma 1 the above equations imply that P_Q is strictly proper.

(ii) Let $P_Q \in \mathbb{R}_{pr}^{p\times m}(s)$ and be strictly proper such that $P_Q \in \psi(C_0)$. Then, according to Corollary 1, P_Q can be written as in (10) for some $Q \in S^{p\times m}$. Now as seen in (9) our hypothesis that P_0 is strictly proper implies that D_1 is biproper, and since also by hypothesis P_Q is strictly proper, from Lemma 1 and (10) it follows that we must have

$$(B_1 - QD_1)(\infty) = 0 \tag{13}$$

which holds true iff Q is strictly proper. □

In order to determine Q we proceed as follows. Let $P_Q = B_Q A_Q^{-1}$ with $B_Q \in S^{p\times m}$, $A_Q \in S^{m\times m}$ right coprime in $\bar{\Omega}$. Now $P_Q \in \psi(C_0)$ implies that

$$D_1 A_Q + N_1 B_Q =: U \in S^{m\times m} \tag{14}$$

is an S-unimodular matrix. Also, according to the above we must have that

$$P_Q = B_Q A_Q^{-1} = (A_1 + QN_1)^{-1}(B_1 - QD_1) \tag{15}$$

for some $Q \in S^{p\times m}$. Solving (18) with respect to Q we obtain the expression for Q:

$$Q = (B_1 A_Q - A_1 B_Q)U^{-1} \tag{16}$$

which is clearly strictly proper since (16) is a right coprime in $\bar{\Omega}$ fractional representation and by assumption both P_0 and P_Q, or equivalently B_1 and B_Q, are all strictly proper (see Lemma 1).

3. Necessary and sufficient condition for robust stability of an additively perturbed plant

Let us now consider the closed-loop unity feedback system $\Sigma_{c1}(P_0, C_0)$ of Fig. 1 where $P_0 \in \mathbb{R}_{pr}^{p\times m}(s)$ is strictly proper and $C_0 \in \phi(P_0)$, and let us make the following assumptions.

(i) The nominal plant P_0 is additively perturbed to $P_0 + \Delta P_0 =: P \in \mathbb{R}_{pr}^{p\times m}(s)$ where $\Delta P_0 \in \mathbb{R}_{pr}^{p\times m}(s)$ strictly proper and known.

(ii) The additively perturbed system $\Sigma(P)$ is also free of unstable hidden modes and $|I_p + [P_0(s) + \Delta P_0(s)]C_0(s)| \not\equiv 0$ as a rational function, i.e. that the perturbed closed-loop system is 'well posed'.

If we now consider the closed-loop unity feedback system $\Sigma_{c1}(P_0 + \Delta P_0, C_0)$ of Fig. 2 then in view of the parametrization in (6') of the set of 'plants' stabilizable by $C_0 \in \phi(P_0)$ we can state the following.

Proposition 3

Under the above assumptions and notation the following statements are equivalent:

(i) $\Sigma_{c1}(P_0 + \Delta P_0, C_0)$ is internally stable and $H_{yu}(P_0 + \Delta P_0), C_0) \in S^{(p+m)\times(p+m)}$

(ii) The additively perturbed plant $P := P_0 + \Delta P_0$ is stabilizable by $C_0 \in \phi(P_0)$

(iii) $P := P_0 + \Delta P_0 \in \psi(C_0)$

(iv) There exists a strictly proper $Q \in S^{p\times m}$ such that

$$P := P_0 + \Delta P_0 = (A_1 + QN_1)^{-1}(B_1 - QD_1) = (B_2 - D_2Q)(A_2 + N_2Q)^{-1} \tag{17}$$

Solving (17) with respect to Q we obtain the following necessary and sufficient condition for the robust stability of an additively perturbed plant.

Theorem 1

Let $P_0 = A_1^{-1}B_1 = B_2A_2^{-1} \in \mathbb{R}_{pr}^{p\times m}(s)$ and strictly proper, $C_0 = D_1^{-1}N_1 = N_2D_2^{-1} \in \mathbb{R}_{pr}^{m\times p}(s)$ a pair of nominal plant and nominal stabilizing compensator so that (3) is satisfied. Let P_0 be additively perturbed to $P_0 + \Delta P_0 =: P \in \mathbb{R}_{pr}^{p\times m}(s)$. Then $\Sigma_{c1}(P_0 + \Delta P_0, C_0)$ is internally stable iff

$$\begin{aligned} Q &:= -D_2^{-1}[(P_0 + \Delta P_0)C_0 + I_p]^{-1}\Delta P_0 A_2 \\ &= -A_1\Delta P_0[C_0(P_0 + \Delta P_0) + I_m]^{-1}D_1^{-1} \in S^{p\times m} \end{aligned} \tag{18}$$

and is strictly proper.

Example

Consider an SISO system with a 'high frequency' model P where $P = 100/(s-1) \times (s+100)$ and a 'low-frequency' model of the plant is $P_0 = 1/(s-1)$. Obviously pure gain feedback can stabilize P_0, e.g. $C_0 = 2$ is a stabilizing compensator for P_0. Now

$$\Delta P_0 = P - P_0 = \frac{-s}{(s-1)(s+100)}$$

Also P_0 can be written as

$$P_0 = \frac{1}{s+a}\left[\frac{s-1}{s+a}\right]^{-1} = BA^{-1}, a > 0$$

and (A, B) are coprime in $\bar{\mathbb{C}}^+$ so that from criterion (18)

$$Q = -\left[\frac{100}{(s-1)(s+100)}2 + 1\right]^{-1}\frac{-s}{(s-1)(s+100)}\frac{s-1}{s+a} = \frac{s(s-1)}{(s^2 + 99s + 100)(s+a)} \in S$$

and is strictly proper. Thus $C_0 = 2$ also 'stabilizes' the 'high-frequency' model P.

Starting from (17) we can also give a parametrization of the set $\Delta\Pi$ of all allowable perturbations ΔP_0 of a strictly proper plant P_0 such that if C_0 stabilizes P_0 then C_0 also stabilizes $P_0 + \Delta P_0$. A similar result to the one below has also appeared in Huang and Lin (1987).

Proposition 4

Let $P_0 \in \mathbb{R}_{pr}^{p\times m}(s)$ be strictly proper with $P_0 = A_1^{-1}B_1 = B_2A_2^{-1}$ and $C_0 \in \phi(P_0)$ with $C_0 = D_1^{-1}N_1 = N_2D_2^{-1}$ so that (3) is satisfied. Then the set of all additive perturbations ΔP_0 such that C_0 also stabilizes $P_0 + \Delta P_0$ is given by

$$\Delta\Pi = \{\Delta P_0(Q) = -(A_1 + QN_1)^{-1}QA_2^{-1} \mid Q \in S^{p\times m} \text{ and strictly proper}\} \tag{19}$$

Proof

Solving (17) with respect to ΔP_0 we have

$$\begin{aligned}\Delta P_0 &= (A_1 + QN_1)^{-1}(B_1 - QD_1) - B_2A_2^{-1} = (A_1 + QN_1)^{-1} \\ &\quad \times [(B_1 - QD_1)A_2 - (A_1 + QN_1)B_2]A_2^{-1} \\ &= -(A_1 + QN_1)^{-1}Q(D_1A_2 + N_1B_2)A_2^{-1} = -(A_1 + QN_1)^{-1}QA_2^{-1}\end{aligned} \tag{20}$$

where we have made use of the facts that $B_1A_2 - A_1B_2 = 0$ and $D_1A_2 + N_1B_2 = I_m$. Notice that every $\Delta P_0 \in \Delta\Pi$ is strictly proper. □

4. Robustness of stability for stable plants

As before, let $P_0 \in \mathbb{R}_{pr}^{p\times m}(s)$ be strictly proper, $C_0 \in \mathbb{R}_{pr}^{m\times p}(s)$ with $P_0 = A_1^{-1}B_1 = B_2A_2^{-1}$, $C_0 = D_1^{-1}N_1 = N_2D_2^{-1}$ so that the Bezout identity (3) is satisfied and consider the internally stable closed-loop system $\Sigma_{c1}(P_0, C_W)$ where $C_W \in \phi(P_0)$ as given by (5) and the non-singularity constraints are automatically satisfied. Now let P_0 be perturbed to $P_0 + \Delta P_0$ where ΔP_0 is strictly proper.

Assuming that $\Sigma(P_0 + \Delta P_0)$ is also free of unstable hidden modes, for C_W to maintain the internal stability of $\Sigma_{c1}(P_0 + \Delta P_0, C_W)$ we must have that (i) $|I_p + [P_0(s) + \Delta P_0(s)]C_W(s)| \not\equiv 0$ and (ii) $P_0 + \Delta P_0 \in \psi(C_W)$ where $\psi(C_W)$ is given by (6). Equivalently $\Sigma_{c1}(P_0 + \Delta P_0, C_W)$ is internally stable iff

$$\begin{aligned}Q &:= -(D_2 - B_2W)^{-1}[(P_0 + \Delta P_0)C_W + I_p]^{-1}\Delta P_0A_2 \\ &= -A_1\Delta P_0[C_W(P_0 + \Delta P_0) + I_m]^{-1}(D_1 - WB_1) \in S^{p\times m}\end{aligned} \tag{21}$$

If $\Sigma(P_0)$ is (open-loop) internally stable then $P_0 \in S^{p\times m}$ (with $\Omega \equiv \mathbb{C}^+$) and we can take $B_1 = B_2 = P_0$, $A_1 = I_p$, $A_2 = I_m$, $D_1 = I_m$, $D_2 = I_p$, $N_1 = Q_{m,p}$, $N_2 = O_{p,m}$ so that $\phi(P_0)$ is given by (Desoer *et al.* 1980, Vidyasagar 1985)

$$\phi(P_0) = \{C_W = (I_m - WP_0)^{-1}W = W(I_p - P_0W)^{-1} \mid W \in S^{m\times p}\} \tag{22}$$

Noticing that the expressions in (22) constitute fractional representations of C_W left and right coprime in $\bar{\Omega}$ and substituting these for C_W in condition (21) we obtain

$$\begin{aligned}Q &= -(I_p - P_0W)^{-1}[(P_0 + \Delta P_0)W(I_p - P_0W)^{-1} + I_p]^{-1}\Delta P_0 \\ &= -(\Delta P_0W + I_p)^{-1}\Delta P_0\end{aligned} \tag{23}$$

and thus we can state the following result.

Proposition 5

Let $P_0 \in S^{p\times m}$ and be strictly proper. Let $C_W = \phi(P_0)$ and let P_0 be additively perturbed to $P_0 + \Delta P_0$ where ΔP_0 is strictly proper and known. Then under condition (i) $\Sigma_{c1}(P_0 + \Delta P_0, C_W)$ is internally stable iff

$$Q := -(\Delta P_0W + I_p)^{-1}\Delta P_0 = -\Delta P_0(W\Delta P_0 + I_m)^{-1} \in S^{p\times m} \tag{24}$$

Now from the fact that Q in (24) gives the closed-loop transfer function matrix of the configuration in Fig. 3, we can rephrase Proposition 5 as follows.

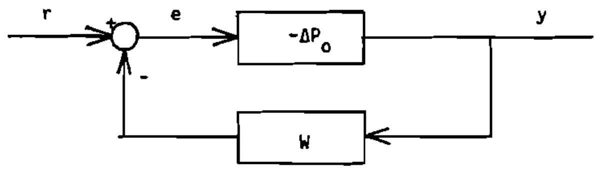

Figure 3.

Proposition 6

Let $P_0 \in S^{p \times m}$ and $P \in \mathbb{R}_{pr}^{p \times m}(s)$ both be strictly proper. Then there exists a compensator C_W stabilizing both, i.e. P_0, P are simultaneously stabilizable iff their difference $\Delta P_0 := P - P_0$ is stabilizable by a proper and Ω-stable compensator W placed on the feedback loop.

Finally, solving (24) with respect to ΔP_0 we obtain the following.

Corollary 2

Let P_0, C_W be as in Proposition 5. Then any ΔP_0 such that $\Sigma_{c1}(P_0 + \Delta P_0, C_W)$ is internally stable must have fractional representations that are left and right coprime in $\bar{\Omega}$ and given by

$$\Delta P_0 = -Q(I_m + WQ)^{-1} = -(I_p + QW)^{-1}Q \tag{25}$$

for some strictly proper $Q \in S^{p \times m}$.

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